Insufficient or Excessive Investment in Economies at Risk of Sovereign Default*

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October 2024

Abstract

Private agents do not internalize the impact of their capital investment decisions on their sovereign's bond prices and risk of default. Thus, a standard externality argument implies that investment is insufficient and a subsidy can improve welfare, if financed by non-distortionary means. We contrast this logic with a countervailing force. When the sovereign is impatient relative to households, plausibly due to Political Economy factors, it finds laissez-faire capital accumulation excessive and might prefer to tax it instead. We embed both mechanisms in a sovereign default model, with decentralized capital investment, long-term public debt, and stochastic trend growth, calibrated to salient features of European periphery economies. We find that the impatience channel dominates quantitatively, to such an extent that laissez-faire is preferable to the government's ideal fiscal policy, based on households' welfare.

Keywords: sovereign default, capital accumulation, decentralized investment
JEL classification: E22, F34, F43

*The views expressed in this paper are those of authors and do not necessarily reflect the views of Bank of Korea. We are grateful to the Ohio Supercomputer Center for access to the Pitzer and Owens clusters.

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This paper explores private capital accumulation behavior in economies where the government implements policy under discretion and can default on outstanding public debt, in the tradition of Eaton and Gersovitz (1981). We find that, as expected based on prior work such as Park (2017) and Gordon and Guerron-Quintana (2018), higher investment generally lowers the probability of default. In turn, this lower risk translates into higher sovereign bond prices now, more favorable borrowing terms and expanded fiscal space for the sovereign. Our analysis concerns the interplay of two mechanisms which determine the fiscal treatment of investment and its desirability.

First, as the private sector does not internalize the impact of its investment choices on public bond prices and default risk, a standard externality argument implies that an investment subsidy, if funded by non-distortionary means, improves welfare. Seoane and Yurdagul (2023) and Esquivel (2024) are two recent papers touching upon this argument. Second, governments in emerging markets and other countries at risk of default seem plagued by Political Economy dysfunctions, incentives that render fiscal authorities effectively more impatient than their constituents. Several works highlight this feature and its role in the quantitative success of sovereign default models, including Alesina and Tabellini (1990), Chatterjee and Eyigungor (2019), Aguiar, Amador, and Fourakis (2020), Azzimonti and Mitra (2023a, 2023b), and Acharya, Rajan, and Shim (2024). We do not take a stand on the exact nature of these frictions but instead emphasize their consequences for capital accumulation. Generally, such a government finds laissez-faire private sector investment excessive, and would like to tax rather than subsidize it, to induce higher consumption immediately. The government being more impatient than domestic households is distinct from the finding in most quantitative work on default that sovereigns are more impatient than their international lenders, a known driver of procyclical fiscal policy (Cuadra, Sanchez, and Sapriza 2010).

We embed these two forces and evaluate their interaction in a quantitative sovereign default model with private investment, random trend growth, and long-term defaultable public debt. We calibrate our model to European periphery economies and use it to disentangle the two forces we study, as well as evaluate the welfare implications of decentralized investment.

We compare our baseline economy, where capital accumulation is decided by the private sector and the government is as impatient as called for by the data, to three counterfactual economies. First, we contrast with an otherwise identical economy in which the government can perfect control investment. This isolates the externality impinging on the sovereign's bond prices and default risk. We then consider a version of our model in which the government is as patient as households, to highlight the impact of this disconnect on the tax treatment of investment in the baseline. Finally, we evaluate a scenario where the government is not impatient and has control over investment.

We find that giving the government access to an appropriately-funded subsidy on investment can indeed improve outcomes, but critically only if the government's discount factor is aligned with that of domestic households. This is not the case in the baseline calibration featuring non-trivial sovereign impatience, to such an extent that, if given the opportunity, our sovereign would want to tax investment, to induce higher consumption immediately. Such a policy significantly worsens household welfare, by about 1% in consumption equivalent measure. In contrast, we find much smaller, albeit positive, welfare gains from introducing the investment subsidy, of about 0.07%, if the sovereign is not relatively impatient. Remarkably, these welfare gains and losses are almost exclusively attributable to changes in default propensity, from capital deepening and sovereign discounting, with little to no changes in business cycle statistics and comovements.

Related literature. Our results connect to several themes previously explored in the literature. With a first group of papers, we share the focus on interactions between capital accumulation and default, including Bai and Zhang (2012) on financial liberalization, Bocola (2016) on the pass-through of sovereign risk, Arellano, Bai, and Mihalache (2018), Esquivel (2024), and Deng and Liu (2024) on sectoral misallocation, Alessandria, Bai, and Deng (2020) on migrant flows, and Asonuma and Joo (2023) on public capital.

Several recent papers have explored ways in which private decision-making constrains discretionary fiscal policy, for example Na et al. (2018) on downward rigid wages and exchange rate regimes, Arellano, Bai, and Mihalache (2020) on pricing frictions and inflation-targeting monetary policy, and Liu and Shen (2022) on frictional labor markets. For us, private sector capital accumulation plays this role.

Finally, our focus on the (de)centralized nature of physical capital investment is distinct but related to the question of (de)centralized international borrowing, as explored by Jeske (2006), Kim and Zhang (2012), and Seoane and Yurdagul (2023), among others.

Roadmap. Section 1 lays out our models, Section 2 highlights qualitative features of first-order conditions for investment under alternative fiscal arrangements, Section 3 compiles our quantitative results, and Section 4 concludes with a brief discussion of our findings and their implications. The Appendix collects supporting material on our data sources, numerical solution, and model detrending.

1 Model

Our model focuses on the interactions between a sovereign fiscal authority, the domestic private sector, and foreign lenders. The model closest to our work is Gordon and Guerron-Quintana (2018), from which we mainly differ by adopting a "the cycle is the trend" productivity process, as in Aguiar and Gopinath (2007). We discuss our agents' problems in turn and then define the equilibrium of this economy. We consider alternative assumptions about the ability of fiscal authorities to shape private investment behavior, as well as its degree of impatience. We start with the laissez-faire case, where the sovereign lacks fiscal instruments that impact investment directly, to then consider the polar opposite case, of perfect public control over capital investment.

The private sector. The private sector in our model consists of a representative household and a representative firm. The household owns shares in the firm and receives dividend payments. In turn, the firm's investment decisions are shaped by the households marginal rate of substitution. Since there are no financial frictions between households and firms, it is often convenient to think of them as a consolidated entity, the private sector.

The household maximizes its expected utility $\mathbb{E}_0 \sum_t \beta^t u(c_t)$ by choosing shareholdings a_{t+1} subject to a sequence of budget constraints given by

$$c_t + P_t a_{t+1} = w_t + (\text{div}_t + P_t)\, a_t + T_t \tag{1}$$

where c_t is the household's consumption, P_t is the stock price, a_t are the shares held at the start of the period, div_t are dividend payments, and T_t is a lump-sum tax or transfer received from the government. We assume, for simplicity, that a unit of labor is supplied inelastically, so that the wage income is simply w_t.[1] As usual with a representative agent, in equilibrium P_t is at a level such that $a_{t+1} = a_t = 1$.

The representative firm hires labor ℓ_t and chooses investment i_t in each period, in order to maximize the present value of dividend payments $\mathbb{E}_0 \sum_t \Lambda_t \text{div}_t$, discounted using the household's pricing kernel $\Lambda_t = \beta^t u'(c_t)/u'(c_0)$, subject to a sequence of the following constraints and laws of motion

$$\text{div}_t = z_t k_t^{\alpha} (\Gamma_t \ell_t)^{1-\alpha} - w_t \ell_t - i_t - \Theta(k_t, k_{t+1}), \tag{2}$$

$$k_{t+1} = (1-\delta) k_t + i_t, \tag{3}$$

1. Our results could extend naturally to the typical case of Greenwood, Hercowitz, and Huffman (1988) preferences, without wealth effects on labor supply, widely used in studies of small open economies. We abstract from elastic labor supply in order to isolate one private sector decision margin alone, investment.

where Θ is a quadratic adjustment cost to investment, a standard feature of production in small open economy models (Schmitt-Grohé and Uribe 2003), and production is constant return to scale, subject to a stationary productivity shock z_t, and a random trend Γ_t. We assume that the stationary "cycle" shock z_t is AR(1), with

$$\log z_{t+1} = (1-\rho_z)\log \mu_z + \rho_z \log z_t + \varepsilon_{z,t},\ \varepsilon_{z,t} \sim \text{ iid } N(0, \sigma_z^2) \tag{4}$$

and the trend accumulates random gross growth rate shocks, $\Gamma_{t+1} = \Gamma_t g_t$, with

$$\log g_{t+1} = (1-\rho_g)\log \mu_g + \rho_g \log g_t + \varepsilon_{g,t},\ \varepsilon_{g,t} \sim \text{ iid } N(0, \sigma_g^2). \tag{5}$$

Following Aguiar and Gopinath (2006), we set the adjustment cost function to

$$\Theta(k_t, k_{t+1}) = \frac{\theta}{2}\left(\frac{k_{t+1}}{k_t} - \mu_g\right)^2 k_t \tag{6}$$

so that investment is costly to the extent to which the growth rate of the capital stock deviates from the average growth rate of this economy, μ_g.

The fiscal authority. The government operates in international financial markets by trading a long-term bond, modeled using a decaying perpetuity (Hatchondo and Martinez 2009). It has the option to default on this obligation, at its discretion. It uses the net proceeds from these external operations, together with a lump-sum tax or transfer, to fund a level of public spending G_t. Its budget constraint is given by

$$(1-D_t)\left[q_t(B_{t+1} - (1-\phi)B_t) - \kappa B_t\right] = G_t - T_t. \tag{7}$$

B_t are the outstanding bond units, B_{t+1} is the bond level chosen for the next period, and the market price of bonds is q_t. κ is a normalization of the debt service payment called for by the B_t units of outstanding debt, while the ϕ parameter controls the maturity structure of the debt. If the sovereign chooses to default today, and sets $D_t = 1$, then no debt service payments are made and the primary deficit $G_t - T_t$ must be zero.

We make the simplifying assumption that G_t is at all times proportional to the productivity trend Γ_{t-1}, rather than a choice variable for the sovereign. This way, public spending is constant in the detrended model, a point to which we return momentarily.

Default is followed by a spell of international financial market exclusion for the sovereign, which ends with probability λ. Eventually, the sovereign regains access to international markets without any outstanding debt.[2] The sovereign's choice to default causes disruptions in domestic production, captured in reduced form by a penalty function applied to the cycle component of productivity, $h(z_t)$, which proxies for the mechanisms explored by Mendoza and Yue (2012) and Arellano, Bai, and Bocola (2024), among others, of trade or banking disruptions caused by sovereign default.

We assume that the government values consumption, much like the household, except possibly for a higher degree of impatience. Its objective is given by $\mathbb{E}_0 \sum_t \beta_g^t u(c_t)$, with $\beta_g \leq \beta$. This assumption captures in reduced form a long tradition of studying Political Economy frictions in sovereign borrowing by emerging markets.

International lenders. Foreign buyers of the government's bonds are competitive and risk-neutral. They have access to a constant short-term risk-free rate r^f, which acts as their opportunity cost of funds. These assumptions deliver the standard long-term bond pricing condition that ensures lenders break even in expectation,

$$q_t = \frac{\mathbb{E}_t(1 - D_{t+1})(\kappa + (1 - \phi)q_{t+1})}{1 + r^f}. \tag{8}$$

This bond pricing is consistent with the assumption of full debt repudiation upon default.

With this maturity structure, the yield-to-maturity of the bond is given by $r_t = \frac{\kappa}{q_t} - \phi$, the spread is $r_t - r^f$, and the bond's risk-free Macaulay duration is $(1 + r^f)/(\phi + r^f)$.

1.1 A small k, big K recursive formulation of laissez-faire

We pursue first a recursive formulation of our environment, where investment is decided by the private sector and the sovereign's choices are limited to its international borrowing and default, i.e., the laissez-faire case. We set up a standard "small k, big K" trick so that firms do not internalize the impact of their individual investment decisions (k') for the aggregate capital stock next period (K') and thus allocations. To economize on notation, we group state variables into two sets: first, exogenous and stationary shocks $s = \langle z, g \rangle$, and second, the endogenous capital and debt stocks, together with the level of the trend in the previous period, $S = \langle K, B, \Gamma_{-1} \rangle$.[3]

The value function for the private sector, the household and firm consolidated, under normal international market access for the sovereign, given that the government will have

2. We assume full repudiation of the defaulted debt. Yue (2010) and Dvorkin et al. (2021) provide reference models with haircuts and recovery, using cooperative and non-cooperative bargaining, respectively.

3. In the Appendix, we discuss "detrended" versions of our models, where Γ_{-1} disappears as a state variable. The detrended models can be then computed using standard methods. This detrending is enabled by the usual homogeneity properties of value and bond price functions, and our functional form assumptions.

B' outstanding debt next period, satisfies

$$v^r(k,s,S,B') = \max_{k'} u(c) + \beta \mathbb{E}_{s'|s} v(k',s',S') \tag{9}$$

$$\text{s.t. } c + k' + \Theta(k,k') = zk^\alpha \Gamma^{1-\alpha} + (1-\delta)k + T(s,S,B') \tag{10}$$

$$K' = \mathcal{K}(s,S,B'), \tag{11}$$

where $\mathcal{K}$ is the perceived law of motion for aggregate capital. In equilibrium, a consistency condition ensures that the capital investment choice of the representative household coincides with the aggregate variable, $k' = K' = \mathcal{K}(s,S,B')$.

The value under default, in international market exclusion, is analogous,

$$v^d(k,s,S) = \max_{k'} u(c) + \beta \mathbb{E}_{s'|s} \Big(\lambda v(k',s',S') + (1-\lambda) v^d(k',s',S') \Big) \tag{12}$$

$$\text{s.t. } c + k' + \Theta(k,k') = h(z)k^\alpha \Gamma^{1-\alpha} + (1-\delta)k + T^d(s,S) \tag{13}$$

$$K' = \mathcal{K}^d(s,S), \quad B' = 0, \tag{14}$$

and the start-of-period value, prior to the sovereign's default decision, satisfies

$$v(k,s,S) = \mathcal{D}(s,S) v^d(k,s,S) + (1-\mathcal{D}(s,S)) v^r(k,s,S,\mathcal{B}(s,S)), \tag{15}$$

where $\mathcal{D}(s,S)$ is the sovereign's equilibrium default policy and $\mathcal{B}(s,S)$ is its choice of bonds for next period. The private sector is impacted by the sovereign's choices through the level of the tax or transfer T and T^d, as well as the productivity penalty h, if in default.

These private sector value functions yield policy function under repayment and default, respectively: the consumption policies $\mathcal{C}(s,S,B')$ and $\mathcal{C}^d(s,S)$, and the capital policies $\mathcal{K}(s,S,B')$ and $\mathcal{K}^d(s,S)$. These policies act as further constraints on the government's choices, since the sovereign understands that, for example, by borrowing more, a counterfactually higher B', the resulting capital stock next period K' will respond, as governed by the $\mathcal{K}$ policy.

The sovereign's value function under repayment is given by

$$V^r(s,S) = \max_{B'} u(\mathcal{C}(s,S,B')) + \beta_g \mathbb{E}_{s'|s} V(s',S') \tag{16}$$

$$\text{s.t. } T(s,S,B') = q(s,S')(B' - (1-\phi)B) - \kappa B - G(S) \tag{17}$$

$$K' = \mathcal{K}(s,S,B'), \tag{18}$$

with associated bond issuance policy $\mathcal{B}(s,S)$, while the value under default satisfies

$$V^d(s,S) = u(\mathcal{C}^d(s,S)) + \beta_g \mathbb{E}_{s'|s}\Big(\lambda V(s',S') + (1-\lambda)V^d(s',S')\Big) \tag{19}$$

$$\text{with } T^d(s,S) = -G(S) \tag{20}$$

$$K' = \mathcal{K}^d(s,S), \quad B' = 0. \tag{21}$$

The start of period value inclusive of the option to default is given by

$$V(s,S) = \max_{D\in\{0,1\}} D\, V^d(s,S) + (1-D)V^r(s,S) \tag{22}$$

and we encode the resulting equilibrium default policy in $\mathcal{D}(s,S)$.

Finally, the bond price schedule in recursive notation satisfies

$$q(s,S') = \frac{1}{1+r^f}\,\mathbb{E}_{s'|s}(1-\mathcal{D}(s',S'))\left[\kappa + (1-\phi)q(s',S'')\right] \tag{23}$$

$$\text{with } S'' = \langle \mathcal{K}(s',S',\mathcal{B}(s',S')), \mathcal{B}(s',S'), \Gamma_{-1}g\rangle. \tag{24}$$

Due to the long-term nature of the bond, its pricing reflects all future equilibrium default, borrowing, and investment policies, at all horizons.

1.2 A recursive formulation of centralized investment

We will contrast the values and policies from the previous section, under laissez-faire, to outcomes under centralized investment, where we allow the government to choose capital investment directly. We illustrate shortly, in the next section, how this can be decentralized using a linear investment tax or subsidy, by means on first-order conditions.

In the centralized case, the value of the government under repayment is given by

$$V_c^r(s,S) = \max_{K',B'} u(c) + \beta_g \mathbb{E}_{s'|s} V_c(s',S') \tag{25}$$

$$\text{s.t. } c + K' + \Theta(K,K') + G(S) = zK^\alpha\Gamma^{1-\alpha} + (1-\delta)K + q_c(s,S')(B' - (1-\phi)B) - \kappa B, \tag{26}$$

and denote the resulting policies by $K' = \mathcal{K}_c(s,S)$ and $B' = \mathcal{B}_c(s,S)$. In default, the choice of capital satisfies

$$V_c^d(s,S) = \max_{K'} u(c) + \beta_g \mathbb{E}_{s'|s} \Big(\lambda V_c(s',S') + (1-\lambda) V_c^d(s',S') \Big) \tag{27}$$

$$\text{s.t. } c + K' + \Theta(K,K') + G(S) = h(z) K^{\alpha} \Gamma^{1-\alpha} + (1-\delta) K \tag{28}$$

$$B' = 0, \tag{29}$$

with associated policy function $K' = \mathcal{K}_c^d(s,S)$.

Finally, the default decision is now

$$V_c(s,S) = \max_{D \in \{0,1\}} D\, V_c^d(s,S) + (1-D) V_c^r(s,S), \tag{30}$$

and the policy is encoded in $\mathcal{D}_c(s,S)$. The lenders' bond pricing condition is analogous to the one under the laissez-fare case, except for the fact that now the relevant policy functions are $\mathcal{D}_c$, $\mathcal{K}_c$, and $\mathcal{B}_c$, respectively, rather than those without the c subscript.

$$q_c(s,S') = \frac{1}{1+r^f} \mathbb{E}_{s'|s} (1 - \mathcal{D}_c(s',S')) \left[\kappa + (1-\phi) q_c(s',S'') \right] \tag{31}$$

$$\text{with } S'' = \langle \mathcal{K}_c(s',S'), \mathcal{B}_c(s',S'), \Gamma_{-1} g \rangle. \tag{32}$$

For the purpose of welfare calculations, we can construct value functions for the private sector, noting that now it makes no decision. For example, under market access,

$$v_c^r(s,S) = u(c) + \beta \mathbb{E}_{s'|s} v_c(s',S') \tag{33}$$

$$\text{with } c + K' + \Theta(K,K') = z K^{\alpha} \Gamma^{1-\alpha} + (1-\delta) K + T_c(s,S,B') \tag{34}$$

$$K' = \mathcal{K}_c(s,S), \quad B' = \mathcal{B}_c(s,S). \tag{35}$$

1.3 Equilibrium

We can now define the equilibria corresponding to these two regimes, with respect to the ability of the sovereign to impact private investment choices.

Definition 1 (MPE Laissez-Faire)**.** A recursive Markov Perfect Equilibrium of the laissez-faire economy consists of

- private sector value functions v^r, v^d, and v, and policies $\mathcal{C}$ and $\mathcal{K}$,
- government value functions V^r, V^d, and V, and policies $\mathcal{D}$ and $\mathcal{B}$, and
- the bond price schedule q,

which together satisfy the following conditions:

- given government policies, the private sector values and policies solve (9) and (12),
- given private sector policies and the bond price schedule, government values and policies solve (16) and (22),
- given policies, lenders break even and the bond price schedule (24) holds.

Definition 2 (MPE Centralized)**.** A recursive Markov Perfect Equilibrium of the centralized investment economy consists of

- government value functions V_c^r, V_c^d, and V_c,
- government policies $\mathcal{D}_c$, $\mathcal{B}_c$, and $\mathcal{K}_c$, and
- the bond price schedule q_c,

which together satisfy the following conditions:

- given the bond price schedule, policies and values solve (25), (27), and (30),
- given policies, lenders break even and the bond price schedule (31) holds.

2 A First-Order Condition Characterization

We start our analysis by comparing the first-order optimality conditions for investment, under laissez-faire and centralized investment, to highlight the forces shaping the optimal size of the capital stock. We focus on investment when the sovereign is choosing to service the debt, although similar conditions can be derived under default as well, and set $\theta = 0$, no adjustment costs, for ease of illustration in this section alone. We revert temporarily to the more compact sequential notation.

Under laissez-faire, the first-order condition for the private sector's capital choice is

$$u'(c_t) = \beta \mathbb{E}_t \left\{ \left[D_{t+1} u'(c_{t+1}^d) h(z_{t+1}) + (1 - D_{t+1}) u'(c_{t+1}^r) z_{t+1} \right] \alpha \left(\frac{\Gamma_{t+1}}{K_{t+1}} \right)^{1-\alpha} + 1 - \delta \right\} \tag{36}$$

while under centralized investment, when the government has perfect control over capital accumulation, the corresponding condition reads

$$u'(c_t)\left[1-\frac{\partial q_t}{\partial K_{t+1}}(B_{t+1}-(1-\phi)B_t)\right]=\beta_g\mathbb{E}_t\Bigg\{ \\ \left[D_{t+1}u'(c^d_{t+1})h(z_{t+1})+(1-D_{t+1})u'(c^r_{t+1})z_{t+1}\right]\alpha\left(\frac{\Gamma_{t+1}}{K_{t+1}}\right)^{1-\alpha}+1-\delta\Bigg\} \tag{37}$$

Two differences are salient. First, the marginal benefit of investment, on the right-hand-side, is evaluated using β_g rather than β. All else equal, future consumption is weakly less valuable today. This discourages investment. Second, the private sector does not internalize the impact of its investment on the bond price received by the sovereign today. This is reflected in the square brackets term on the left-hand-side, scaling the marginal cost of investment.

In equilibrium, and across the state space, the relation between capital levels and default propensity is non-monotonic (Gordon and Guerron-Quintana 2018). Still, in general, for most of the states in the ergodic distribution, more capital means weaker incentives to default, and so the benefit of capital accumulation is underestimated by the private sector investment policy. This social desirability of higher capital is reflected in the $\frac{\partial q_t}{\partial K_{t+1}}>0$ term. A more subtle implication of long-term debt is that when the government is buying back, retiring outstanding bonds in secondary markets, $B_{t+1}<(1-\phi)B_t$, the private sector could invest excessively, in that its high K_{t+1} choice supports a higher bond price, a price that now the sovereign pays rather than receives.

Our quantitative analysis in the following section aims to establish which of these differences in investment behavior dominates and how the answer depends on the nature of the frictions impinging on this economy.

The decentralization tax or subsidy. So far, we laid out two polar opposite cases: laissez-faire, under which the private sector decides on investment and the government issues bonds, and a centralized economy, where the government controls both assets. We now sketch how this centralized outcome can be implemented by means of a state-contingent subsidy to capital accumulation, mirroring the analysis of Gordon and Guerron-Quintana (2018, Appendix A.2). We introduce a subsidy τ_t on capital accumulation in the firm's problem. The definition of dividend payments now becomes

$$\text{div}_t=z_tk_t^{\alpha}\left(\Gamma_t\ell_t\right)^{1-\alpha}-w_t\ell_t+(1-\delta)k_t-(1-\tau_t)k_{t+1}-\Theta(k_t,k_{t+1}) \tag{38}$$

and the problems facing private sector agents are otherwise unaltered. The subsidy also enters the primary deficit expression on the right-hand-side of the government's budget constraint, which now reads

$$(1 - D_t)\left[q_t(B_{t+1} - (1-\phi)B_t) - \kappa B_t\right] = G_t + \tau_t K_{t+1} - T_t. \tag{39}$$

The consolidated private sector's first order condition for K_{t+1} now becomes

$$\begin{aligned}(1-\tau_t)\,u'(c_t) = \beta\mathbb{E}_t\Bigg\{ \\ \Big[D_{t+1}u'(c^d_{t+1})h(z_{t+1}) + (1-D_{t+1})u'(c^r_{t+1})z_{t+1}\Big]\,\alpha\left(\frac{\Gamma_{t+1}}{K_{t+1}}\right)^{1-\alpha} + 1 - \delta\Bigg\}\end{aligned} \tag{40}$$

so that the only difference from (36), under laissez-faire, is the scaling of the marginal cost of investment on the left hand side by the $1-\tau_t$ subsidy term. A comparison with (37), the centralized investment first order condition, delivers the state-contingent subsidy rate which supports the centralized investment policies,

$$\tau_t = 1 - \left[1 - \frac{\partial q_t}{\partial K_{t+1}}(B_{t+1} - (1-\phi)B_t)\right]\frac{\beta}{\beta_g}. \tag{41}$$

Whether τ_t is in fact a subsidy ($\tau_t > 0$) or a tax ($\tau_t < 0$) depends on the three factors. First, the strength of the investment externality, $\frac{\partial q_t}{\partial K_{t+1}}$, second, whether the sovereign is issuing or buying back long-term bond units and how many, $B_{t+1} \lesseqgtr (1-\phi)B_t$, and finally, the degree of sovereign impatience, from $\frac{\beta}{\beta_g} \geq 1$.

In default, an analogous derivation leads us to conclude that, since the externality through q_t is no longer a concern, due to market exclusion, the tax or subsidy reflects potential sovereign impatience alone: $\tau^d_t = 1 - \frac{\beta}{\beta_g}$. If the sovereign is not impatient, it neither taxes nor subsidizes investment during default. If instead the sovereign is impatient, it will tax capital accumulation, without ambiguity.

3 Quantitative Analysis

To fully specify our model and evaluate its quantitative properties, we need to make further assumptions concerning functional forms. We use $u(c) = -c^{-1}$ for the flow utility function, and thus set the coefficient of relative risk-aversion to the conventional value of 2. Our default penalty function follows Chatterjee and Eyigungor (2012), $h(z) =$

$z - \max(0, \iota_0 z + \iota_1 z^2)$, with $\iota_0 < 0 < \iota_1$. This is in line with the observation made by Arellano (2008) that a convex penalty is key for countercyclical spreads and elevated default risk during recessions in this class of models.

3.1 Calibration and model fit

We pursue a quarterly calibration of the model, for the case of decentralized investment, using data from five European periphery countries, Portugal, Italy, Ireland, Greece, and Spain, collectively and informally known as the PIIGS, over the 1995Q1 to 2019Q4 sample. These countries are often the subject of studies concerning the European debt crisis of the mid 2010s. Also convenient for our purposes, these countries are part of the same monetary union, the Euro area, which somewhat preempts potential concerns about the role of monetary policy for our question.

We group parameters in two sets. First, those that can be set directly, based on estimates from the data and values common in the literature. Table 1 compiles their values and interpretation. A second set of parameter values are specific to our model. They are picked to match five moments in the data. Table 2 lists these parameters and the model fit.

	Value	Comment
Fiscal policy and international markets		
G	0.185	Mean public spending to GDP
r^f	0.01	International risk-free real rate
ϕ	0.05	Macaulay duration of debt
λ	0.05	International market exclusion
κ	$\phi + r^f$	Normalization, $q^f = 1$
Production and productivity		
α	0.45	Income shares
δ	0.02	Capital depreciation
ρ_z	0.95	Persistence cycle
ρ_g	0.01	Persistence growth rates
σ_z	0.006	Volatility cycle
σ_g	0.02	Volatility trend
μ_z	0.35	Normalization, detrended $Y = 1$

Table 1: Parameters Set Externally

Our productivity process features a stationary cycle component z_t and a random trend Γ_t driven by potentially persistent growth rates g_t. Estimating such a process on our short sample is challenging. We rely on the GMM method of Aguiar and Gopinath (2007). We estimate the process for each country and then average the estimates, resulting in the parameter values reports in the bottom panel of Table 1. We find that growth rate shocks are essentially uncorrelated over time, although volatile.

The G parameter is set to match the average share of public spending in GDP, α is disciplined by National Accounts factor shares, the depreciation rate for capital δ follows the conventional quarterly value of 2%. The risk-free rate is set to 1% quarterly, based on a longer sample of German bond yields, a value comparable to risk-free rates in the literature, albeit somewhat high compared to recent levels of yields. The Macaulay duration of the debt is set to 5 years, the reference value from Chatterjee and Eyigungor (2012), somewhat lower than the values reported by Bocola and Dovis (2019) for Italy. The average length of market exclusion following default is governed by λ, which we set based on the evidence reported by Tomz and Wright (2013), roughly five years. The debt service payment parameter κ is fixed at $\phi + r^f$, so that the risk-free long-term bond price is one. This normalization has the desirable property of expressing B in units of GDP. Finally, we set μ_z as to normalize mean GDP in the detrended model to unity. Table 4 in the Appendix compiles relevant country-level values.

	Value	Target	Data	Model
β	0.978	Mean investment to GDP	0.21	0.21
β_g	0.973	Corr spread and GDP	−0.22	−0.31
ι_0	−1.00	Mean spread	2.39	2.40
ι_1	1.08	St dev of spread	2.08	1.08
θ	0.65	Relative volatility of investment	4.16	4.16

Table 2: Parameters Set Internally

Five parameters allow us to target five key moments in the data. Nearly all parameters impact most statistics to at least a certain degree, but some parameters are more influential than others, and therefore we align them in Table 2 with the moment that is most sensitive. The investment adjustment cost θ is set to match the volatility of investment relative to that of GDP, roughly four times more volatile. The default penalty parameters ι_0 and ι_1 drive the mean and volatility of yield spreads, in the data about 2.4% and 2%, respectively. As standard in the growth model, the private sector discount factor β controls the capital-income ratio, equivalently the ratio of investment to GDP, about 20%. In turn, the sovereign's discount factor β_g is then most important for the countercyclicality of spreads. We find that β_g is lower than β by about 0.005 in our quarterly calibration, a nontrivial degree of relative impatience for the fiscal authority. The model is largely successful in this moment matching exercise, with one notable exception. It has a hard time delivering the volatility of spreads in the data, with a standard deviation of about 1% in the model versus the 2% figure from the data.

3.2 Findings

Table 3 compiles our results. It reports key statistics from the data, our model calibration, and model-based counterfactuals aiming to decompose the relative contribution of the two frictions we study. Our baseline model, which we calibrated in the previous section, is the "Impatient ($\beta_g < \beta$)" + "Laissez-faire" case. The model exhibits countercyclical spreads, an acyclical trade balance, together with volatilities and a debt-to-GDP ratio comparable to the data.

The third column of Table 3 allows us to explore the consequences of giving the sovereign direct control over the capital accumulation decision or, equivalently, under a rich enough set of fiscal instrument. This internalizes the externality induced by investment on the bond price schedule. On the other hand, all policies in this economy are shaped by the government's discount factor β_g and the private sector's β is only relevant for assessing welfare. The net result in a reduction in the average level of the capital stock, by about 9%, a higher default risk, and an increase in average spreads of about 85 basis points. Spread volatility increases as well, by about 20%, while other moments are mostly unaffected.

We use a standard consumption equivalent measure[4] of welfare gain to assess the value to the household from centralizing investment, moving from our baseline to this counterfactual setting where investment is under the control of the government as well. We find that households are *worse off*, with roughly a 1.03% drop in consumption equivalent welfare. Households would prefer not to give the sovereign the opportunity to internalize the externality, due to its relative impatience.

The last two columns of Table 3 report results under the assumption that public decision-making mirrors the households' discount factor precisely, $\beta_g = \beta$, e.g., the absence of Political Economy frictions. From the fourth column, we learn that eliminating the sovereign's impatience lowers spreads and default risk, while leaving most other moments unchanged. The household is *better off* with such a patient sovereign, and consumption equivalent welfare increases by 0.23%. Even though both regimes suffer from the investment externality, allocations are improved due to the reduced willingness of the government to risk costly default episodes.

4. We evaluate welfare in the state at which shocks are at their mean levels, and debt and the capital stock are at mean in the baseline model's ergodic distribution. Welfare rankings are similar elsewhere in the state space. We use the household's value function, and therefore its discount factor, for this calculation. $c_{\text{eq}} = u^{-1}((1-\beta)v^r)$ and report the percent change in this implied consumption measure across models.

Moment	Data	Impatient ($\beta_g < \beta$)		Patient ($\beta_g = \beta$)	
		Laissez-faire	Centralized	Laissez-faire	Centralized
Ratios to GDP					
Debt	0.37	0.30	0.31	0.30	0.30
Capital stock	—	2.67	2.44	2.67	2.74
Consumption	0.58	0.59	0.59	0.59	0.59
Investment	0.21	0.21	0.19	0.21	0.22
Spread and default (%)					
Mean spread	2.39	2.40	3.24	2.23	2.01
St dev spread	2.08	1.07	1.28	1.01	0.95
Default risk	—	2.13	2.79	1.97	1.80
Standard deviations, relative to GDP					
Consumption	1.00	1.27	1.31	1.27	1.28
Investment	4.16	4.16	4.25	4.34	4.48
Correlations with GDP					
Consumption	0.80	0.96	0.95	0.95	0.95
Investment	0.64	0.30	0.30	0.29	0.28
Trade balance	−0.06	0.00	0.00	0.00	0.00
Spread	−0.22	−0.31	−0.29	−0.32	−0.31
Household welfare (%)					
vs laissez-faire			−1.03		+0.07
vs impatient				+0.23	

Table 3: Model Statistics

NOTES: The "Data" column is the average over the moments of our five countries, reported individually in Table 4, in the Appendix. Our calibrated model corresponds to the "Impatient ($\beta_g < \beta$)" + "Laissez-faire" column. Model moments are computed based on long simulations, 10^6 periods, excluding default spells and the first five years following return to market. The measure for welfare is described in footnote 4.

Finally, we can best highlight the pure value of internalizing the externality by comparing the fourth and fifth columns. In both of these cases the sovereign is patient and the only difference between them is that in the fifth column the government can control investment directly. Capital levels are higher and spreads fall by an additional 20 basis points on average. By internalizing the externality, fiscal policy makes households *better off*, welfare increases by 0.07%.

4 Discussion

Our results imply that the potential benefit from internalizing the investment externality, via an appropriately-designed and funded subsidy, is swamped by the sovereign's preference for immediate consumption, due to its lower discount factor, a classic proxy for Political Economy frictions. What is to be done? Aguiar, Amador, and Fourakis (2020) consider the gains from outright banning international market access for the sovereign and find them to be potentially large, in an endowment model. Less radical, Hatchondo, Martinez, and Roch (2022) propose fiscal rules, institutional limits on the sovereign's ability to borrow into unfavorable bond prices, by means of either a debt of spread break. Azzimonti and Mitra (2023a, 2023b) tackle political constraints and their fiscal consequences directly, in a microfounded model of legislative bargaining over tax and spending outcomes, as well as sovereign default. This broader research agenda points to a need to better understand the exact nature and causes of sovereigns' shortsightedness. Our results call into question the standard prescription according to which households can be made better off through the introduction of targeted incentives for capital accumulation.

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A Data Sources

Our sample consists of the PIIGS countries (Portugal, Italy, Ireland, Greece, and Spain) over the 1995Q1 to 2019Q4 sample. We obtain data on debt-to-GDP, yields and spreads from the European Central Bank (ECB) and National Accounts data from EuroStat. Table 4 compiles data moments for each country, together with the source and the cross-country averages we target in our calibration.

B Model Detrending

Our models feature a random trend, encoded in the Γ_{-1} state variables. Our detrending strategy is largely adapted from Aguiar and Gopinath (2007). We guess and verify that quantities are linear in Γ_{-1} and values are proportional to the inverse of Γ_{-1}, under our assumption of a coefficient of risk-aversion of 2. We denote by $\hat{}$ the detrended variables and values. For any quantity x (consumption, investment, etc.) we have therefore $x = \hat{x}\Gamma_{-1}$, and for any value function v we write $v = \hat{v}/\Gamma_{-1}$. The bond price function is homogeneous

	Mean	Greece	Ireland	Italy	Portugal	Spain	Source
b/y	0.373	n.a.	0.337	0.409	0.510	0.266	ECB
c/y	0.582	0.676	0.381	0.604	0.601	0.603	EuroStat
i/y	0.215	0.187	0.265	0.199	0.201	0.217	EuroStat
g/y	0.182	0.205	0.152	0.190	0.184	0.182	EuroStat
spread	2.390	5.137	1.662	0.658	2.443	1.173	ECB
σ_{spread}	2.077	4.738	1.121	0.672	1.929	1.311	ECB
σ_c/σ_y	1.004	1.079	0.705	0.997	1.117	1.121	Eurostat
σ_i/σ_y	4.158	4.898	6.916	2.898	3.514	2.562	Eurostat
$\sigma_{tb/y}/\sigma_y$	2.167	1.763	3.910	1.078	2.070	2.013	Eurostat
$\rho_{y,c}$	0.798	0.888	0.463	0.801	0.900	0.937	EuroStat
$\rho_{y,i}$	0.641	0.573	0.180	0.823	0.779	0.851	EuroStat
$\rho_{y,tb/y}$	-0.059	-0.254	0.061	0.072	-0.115	-0.058	EuroStat
$\rho_{y,\text{spread}}$	-0.221	-0.520	-0.274	0.082	-0.174	-0.218	EuroStat
σ_y	2.051	2.45	3.07	1.45	1.57	1.72	EuroStat
$\sigma_{\Delta y}$	1.324	1.58	2.90	0.70	0.79	0.65	EuroStat
$\rho_{y_t,y_{t-1}}$	0.815	0.88	0.60	0.81	0.87	0.91	EuroStat
$\rho_{\Delta y_t,\Delta y_{t-1}}$	0.372	0.32	-0.21	0.51	0.42	0.82	EuroStat

Table 4: Data Moments by Country and Average

NOTES: "spread" is the difference in yield-to-maturity between each country's bond and that of Germany, as reported by the ECB. σ denotes standard deviation, and ρ correlation. Debt-to-GDP data is not available for Greece. When appearing in levels, and not as ratios or log differences (Δ), variables are Hodrick and Prescott (1997) filtered with $\lambda = 1600$.

of degree zero in Γ_{-1}, B', and K' and can be written as a function of $\hat{B}'$ and $\hat{K}'$ directly. Finally, our functional form assumption for the investment adjustment cost Θ is consistent with this detrending strategy.

Detrending the model produces small alterations to the resource and budget constraints, as all future stock variables are scaled by the current gross growth rate shock g. For example, the resource constraint for the sovereign, under centralized investment, becomes

$$\hat{c}_t + \hat{K}_{t+1} g_t + \hat{\Theta}_t + \hat{G} = z_t (\hat{K}_t)^{\alpha} (g_t)^{1-\alpha} + (1-\delta)\hat{K}_t + q_{c,t}(\hat{B}_{t+1} g_t - (1-\phi)\hat{B}_t) - \kappa \hat{B}_t. \quad (42)$$

The final necessary adjustment concerns discounting. The effective discount factor in the detrended model is now β / g_t or β_g / g_t, respectively.

C Numerical Solution

The computation of models with defaultable long-term debt is notoriously challenging (Chatterjee and Eyigungor 2012; Gordon and Guerron-Quintana 2018). We tackle this task in our model with capital investment by employing discrete choice methods. See Dvorkin et al. (2021) and Mihalache (2020) for early uses of these methods for default models, and Mihalache (2024) for a pedagogical encyclopedia article.

Our method involves augmenting the default ($\mathcal{D}$) and borrowing ($\mathcal{B}$) decisions with Extreme Value Type I taste shocks. We set the variance of these shocks to the smallest value consistent with convergence. The capital investment choice ($\mathcal{K}$) is not perturbed. Instead, we rely on root-finding on the private sector's first-order condition to find the preferred investment level, given any arbitrary B' borrowing level. In the centralized model, we again find the best K' for each possible B' using bisection and linear interpolation. We solve all versions of the model using the same grids. We discretize the shock process using 9 points for z, 7 for g, 200 for B, and 150 points for K, and allow interpolation over the K dimension. We experimented with grid sizes and location, to confirm the robustness of welfare measures and business cycle statistics.