

The Consequences of Financial Center Conditions for Emerging Market Sovereigns*

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Abstract

Tight financial conditions in international lenders' home markets are broadly perceived to be recessionary and to increase the default risk of emerging markets. This paper evaluates theoretically and quantitatively the degree to which incomplete markets sovereign default models can replicate these patterns from the data. We emphasize the case of pre-announced or predictable interest rate movements in the financial center, for example, at the start of a US monetary policy tightening cycle. An otherwise standard default model, once augmented with a lenders' stochastic discount factor with news shock and domestic financial frictions, robustly exhibits the recessionary effects of high risk-free rates, yet the behavior of spreads is state- and shock-contingent, in line with our theoretical analysis and prior work. This is the case even though in our model capital inflows are expansionary. We argue that the sovereign's willingness to sharply cut borrowing in the face of temporarily higher yields drives the subtle behavior of spreads during such global episodes.

Keywords: global financial cycle, expansionary capital inflows, news shocks, sovereign default
JEL classification: F34, F41, E52

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1 Introduction

This paper studies the consequences of changes in financial center interest rates for emerging market sovereigns, with an emphasis on the case where such movements are pre-announced and persistent, for example at the start of a US monetary policy tightening cycle. The literature has broadly conclude that high interest rates in the US, including from tight monetary policy, are associated with a heightened risk of recession and higher spreads in emerging markets, as exemplified by Uribe and Yue (2006) and, more recently, by Kalemli-Özcan (2019). Figure 1 lays out one of the events motivating our work. We plot the daily spread for the entire EMBI+ emerging markets portfolio, during the “taper tantrum” episode of 2013. On the 22nd of May, then Federal Reserve Chairman Bernanke suggested that tapering of the ongoing Quantitative Easing program is imminent, causing the start of a “taper tantrum” in US bond markets. On June 19th, the Federal Reserve Board announced officially the unwinding of the program, quickly leading to a further increase in the spreads of emerging markets. This episode illustrates both the magnitude of the impact on spreads but also, critically, that changes in financial center conditions are often expected or pre-announced.

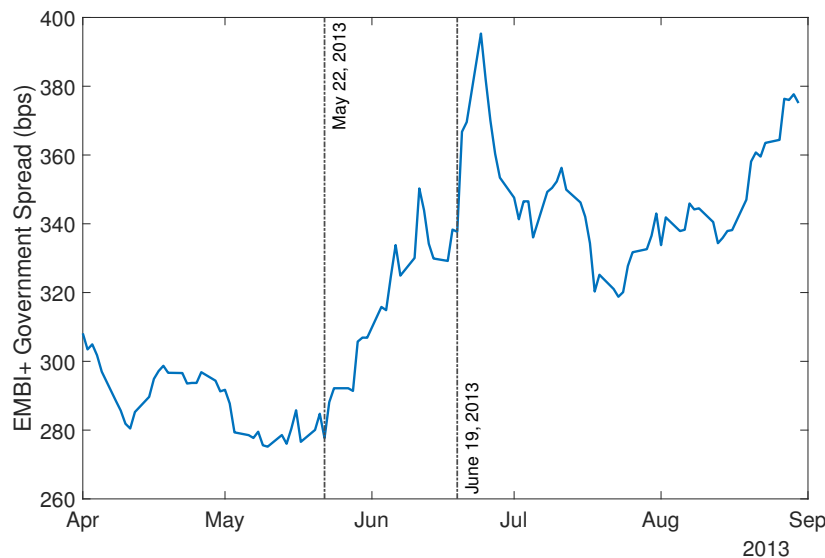


Figure 1: EMBI+ Spread, 2013 “Taper Tantrum” Episode

We make two related contributions. First, we study a highly tractable default model, a setting in which we can characterize tightly optimal issuance and default behavior, and document two countervailing incentives concerning equilibrium spreads. Spreads fall, if increases in the risk-free rate are highly transitory. If, instead, lenders’ opportunity cost of funds is expected to change in the future but not altered today, spreads increase immediately. Finally, persistent and immediate increases in risk-free rates have an ambiguous effect. We characterize the trade-offs shaping these patterns, which put in sharp contrast the limitations of standard models. Second, we propose

a sovereign default model augmented with a rich stochastic discount factor for international lenders, featuring both persistent and expected dynamics, and domestic production subject to financial frictions. We show that in our settings, tight financial conditions in the center are robustly recessionary for the emerging market, but that the response of spreads is state- and shock-contingent.

Outcomes in our model are shaped by the interplay of three interest rates: the yield of the risk-free rate bond in the financial center, the yield of the defaultable bond issued by the sovereign, and the domestic interest rate charged for working capital loan in the emerging market. Following standard practice, we assume that the sovereign alone has access to international markets, with an option to default, and taxes or transfers lump-sum the net proceeds from these international operations to domestic households. In turns, these households supply labor and consume. Their intertemporal marginal rate of substitution induces the interest rate that domestic financial intermediaries charge for working capital loan took by local goods producers. The behavior of this domestic rate is of independent interest, for the broader class of default models. One important feature of this domestic environment is that, in our model, capital inflows are expansionary.

We use data on the US yield curve and inflation to estimate a one-factor pricing kernel with news shocks. A renewal process governs the time-varying mean of the sole factor driving the stochastic discount factor, inducing expected transitions in yields. Once fit to US data, this pricing kernel is employed in our quantitative sovereign default model, to study the consequences of expected and unexpected tight lending conditions for sovereign borrowing, risky yields, and real activity levels.

Related Literature. Earlier contribution such as Neumeyer and Perri (2005) and Uribe and Yue (2006) studied both theoretically and empirically the role of interest rate shocks for the behavior of small open economies, but abstracted from endogenous default risk and pre-announced or predetermined interest rate movements. Fernández-Villaverde et al. (2011) document the importance of stochastic volatility and Johri, Khan, and Sosa-Padilla (2020) revisit this feature in a model with endogenous sovereign default. Wolf and Zessner-Spitzenberg (2021) study the consequences of interest rate hikes in a monetary union, for a two-goods economy with downward rigid wages, where tight monetary policy is recessionary and leads to high spreads. In contrast with these works, we focus on news about the near-term path of interest rates in the financial center and the role domestic financial frictions play in translating these rate movements into recessionary pressures, as well as the sovereign's ability to use borrowing to mitigate their impact.

Our topic also relates to the broader question of international comovement, the Global Financial Cycle, as discussed by Longstaff et al. (2011), Bai, Perri, and Kehoe (2019), Miranda-Agrippino and Rey (2021), and Morelli, Ottonello, and Perez (2022), among others. This line of research stresses the role of a small number of global factors in explaining the cross-section and comovement of spreads. Other spillover channels from which we abstract include lenders' risk-aversion and

wealth effects, as in Lizarazo (2013) and Arellano, Bai, and Lizarazo (2017).

Our stochastic discount factor for international lenders features news shock about future real rates. News shocks were studied extensively in applications to total factor productivity, both theoretical and empirical, in Beaudry and Portier (2004, 2006) and more recently Görtz, Tsoukalas, and Zanetti (2016). We abstract from news about productivity and instead allow only early warning about future interest rate movements in the financial center, leaving the question of their interaction for future work.

Finally, our model exhibits robustly expansionary capital inflows. Whether and how small open economy models can deliver such behavior, and the extent to which the data supports such a stylized fact, has been the subject of an extensive literature, for example Blanchard et al. (2016).

2 Risk-free Rate Movements in a Tractable Default Model

In this section,¹ we characterize the consequences of risk-free rate movements in a highly tractable model, where optimal issuance and default behavior are summarized by a simple shock threshold and a first-order condition. This setting allows us to highlight the key role of interest rate persistence for the dynamics of sovereign spreads and the inherent limitation of models without domestic frictions.

The Sovereign's Problem. A risk-neutral sovereign receives a constant endowment \bar{y} and trades a one-period bond with risk-neutral, competitive lenders, with an option to default of stochastic value. There is no recovery and market exclusion is permanent. In standard recursive notation, the sovereign's value function is

$$V(b|r, r') = \max_{b'} \left\{ \bar{y} - b + q(b'|r, r')b' + \beta \mathbb{E}_v \max \left\{ V(b'|r', r'), V^d - v \right\} \right\} \quad (1)$$

where V^d is a constant and v is an iid shock with pdf ϕ and cdf Φ . The consumption level induced by borrowing b' is given by $c = \bar{y} - b + q(b'|r, r')b'$. The only state variable for the value with market access is the outstanding debt level b , but we are explicit about the dependency on the risk-free rate, this period (r) and in the future (r'), which we treat as non-stochastic.

Default and Bond Prices. The default policy is characterized by a threshold value for v , $v^*(b|r) \equiv V^d - V(b|r, r)$, so that the sovereign default whenever the start-of-period debt level is b and the realization of the iid shock v is below $v^*(b|r)$. The resulting bond price schedule,² consistent with

1. The development of this section has benefited greatly from Yan Bai's careful suggestions, for which we are particularly thankful.

2. The corresponding yield-to-maturity spread is given by $\text{spread}(b'|r, r') = (1+r) \frac{\Phi[v^*(b'|r')]}{1-\Phi[v^*(b'|r')]}$.

lenders breaking even in expectation, is then

$$q(b'|r, r') = \frac{1 - \Phi[v^*(b'|r')]}{1 + r}. \quad (2)$$

Issuance Behavior. Given the threshold form of the default policy, the value function can be expressed as

$$V(b|r, r') = \bar{y} - b + \max_{b'} \left\{ q(b'|r, r')b' + \beta \left[\int_{-\infty}^{v^*(b'|r')} (V^d - v) d\Phi + \int_{v^*(b'|r')}^{\infty} V(b'|r', r') d\Phi \right] \right\} \quad (3)$$

and the associated first-order condition for b' is

$$q(b'|r, r') + b' \frac{\partial q(b'|r, r')}{\partial b'} = \beta [1 - \Phi(v^*(b'|r'))] \quad (4)$$

or, by substituting out the equilibrium q schedule from (2),

$$h(v^*(b'|r')) b' = 1 - \beta(1 + r) \quad (5)$$

where $h(v) \equiv \phi(v) / [1 - \Phi(v)]$ is the hazard function of the v shock.

No Transition Dynamics. Since $V(b|r, r')$ is affine in b with slope -1 , the default threshold $v^*(b|r)$ is affine in b with slope 1. The first-order condition (5) is independent of b so that the debt levels jumps and remains at a “pseudo steady-state” level b_{SS} until the country eventually default, with constant probability $\Phi(v^*(b_{SS}|r))$. For $\beta(1 + r) < 1$, the relevant case for us, in equilibrium the sovereign rolls over a debt position, $b' > 0$, as the hazard function is weakly positive, $h(v) \geq 0$.

We can now evaluate the consequences of changes in the risk-free rate, today (only r), expected (only r'), and permanently (both r and r'), for borrowing and default risk. We start by fixing the risk-free rate at all times to a set level, $r = r' = \bar{r}$, and consider deviations from \bar{r} in turn.

A Fully Transitory Shock. We start with the case in which only this period’s r increases.

Proposition 1 (One-Time, Unexpected Shock). *If the distribution of v has a weakly increasing hazard function h , a one-time, unexpected increase in r above \bar{r} , without any change in future rates, $r' = \bar{r}$, reduced borrowing, default risk, and the spread.*

Proof. See Appendix A. □

When r increases unexpectedly now, but returns to its initial value permanently starting next period, the right-hand side of equation (5) decreases, and thus $h(v^*(b'|\bar{r}))b'$ must decrease as well. Since next period’s r' returns to the pre-shock level \bar{r} , the default threshold function $v^*(b|\bar{r})$ next period is unaltered by today’s one-time change in r . Given the weakly increasing hazard

function³ h , the new solution of the first-order condition must yield a lower b' , a lower $v^*(b'|\bar{r})$, and therefore a lower default risk $\Phi(v^*(b'|\bar{r}))$ and spread.

The Value of Market Access. What about more persistent movements in the risk-free rate? We will consider, for tractability, the extreme case of an unexpected and permanent increase in the risk-free rate, both r and r' change from \bar{r} to $\hat{r} > \bar{r}$. We argue, first, that such a change will lower the value of having access to international financial markets, thus lowering $V(b|\hat{r}, \hat{r})$ at any $b > 0$ level. This is because the cost of servicing the outstanding debt is higher and therefore the default option is weakly more attractive, lowering the v^* threshold. [TODO comp static at steady state]

An Expected Shock. Consider now the case under which r stays at \bar{r} today but we receive news that all future rates r' increase to $\hat{r} > \bar{r}$.

Proposition 2 (An Expected, Permanent Shock). . . . *An expected increase in r' above \bar{r} , without any change in the current rate, $r = \bar{r}$, reduced borrowing and increases the default risk and spread.*

Proof. See Appendix A. □

Without a change in r , the right-hand side of (5) is unaltered. At the old b_{SS} , $v^*(b_{SS}, \hat{r})$ is higher, due to the lower value of ongoing market access when rates are permanently higher, and therefore $h(v^*)$ would be higher. For the first-order condition to hold, b' must fall below b_{SS} . [TODO must fall less than what is needed to keep spread/def pr same?]

An Immediate and Permanent Shock. With a permanent change in both r and r' , the impact on spreads is ambiguous, depending on which of two forces dominates: an incentive to lower borrowing, as in the case of the transitory change, and the permanently lower value of market access, which tends to increase default propensity and therefore spreads. This is because the new risk-free rate alters the value function's intercept and therefore shifts the intercept of the default threshold function $v^*(b|r)$ as well.

Numerical Example. Consider a numerical example plotted in Figure 2, for $\beta = 0.85$, $\bar{y} = 1$, v Normal with mean 0 and standard deviation 2.5, with a reference \bar{r} of 2%. We plot our 3 cases of interest: a one-time, transitory change ($r \neq \bar{r}, r' = \bar{r}$), an expected change ($r = \bar{r}, r' \neq \bar{r}$), and an immediate, permanent change ($r = r' \neq \bar{r}$). As we vary the relevant risk-free rate on the horizontal axes, the choice of borrowing is always lower, in the right (b) panel, but the overall impact for the one-period ahead default probability (and thus the spread) is markedly different across cases. With a one-time change, consistent with Proposition 1, default risk falls monotonically in the r level. If increase the change is expected, from next period onward, spreads increase. Finally,

3. Note that we have restricted attention to shock distributions with a weakly increasing hazard function, such as, among others, the Uniform and the Normal distributions. See Baricz (2008). This is not the case, for example, for the Pareto distribution.

immediate permanent changes yield non-monotonic responses of default risk, it first increases then eventually decreases, depending on which of the two forces dominates.

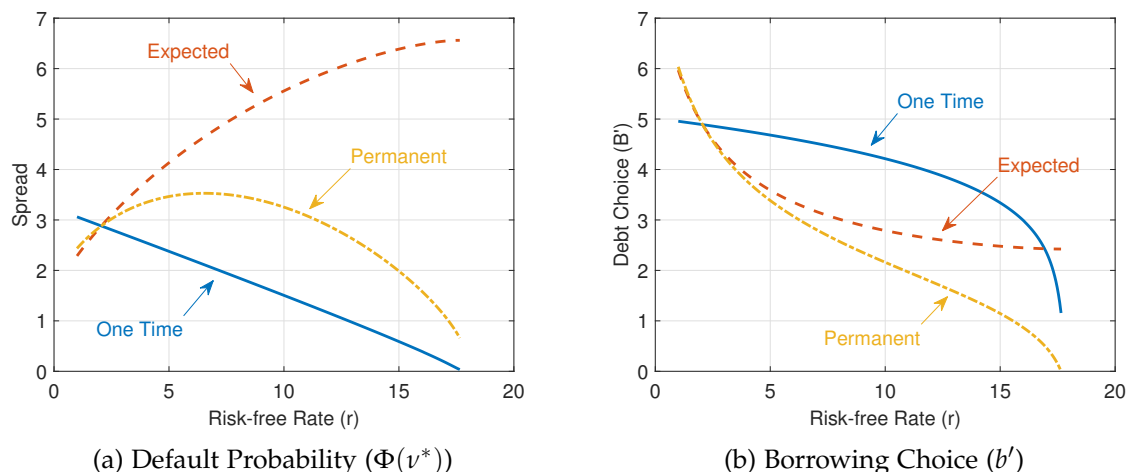


Figure 2: One-Time, Expected, and Permanent Changes in Risk-free Rates

Taken together, these findings suggest that a standard endowment default model can exhibit either higher or lower spreads in response to increases in risk-free rates, depending on shock persistence and predictability.⁴ Given the limitation of the standard endowment approach and the ambiguous effect explored in this section, we pursue a complementary mechanism based on distorted domestic production, with the aim of rationalizing not only the response of spreads to rate changes in the financial center but also domestic income and financial conditions.

3 A Parsimonious Pricing Kernel with News

Our theoretical analysis of a tractable model in Section 2 suggests that shock persistence and predictability are key determinants of the response of spreads to movements in financial center conditions. To explore this question in a quantitative framework, we extend the one-factor, baseline pricing kernel of Vasicek (1977) to allow for jumps and predictable transition dynamics, while economizing on state variables. We describe our pricing kernel and its estimation in turn. We then embed the resulting real pricing kernel in a sovereign default model with domestic financial frictions, in the following section.

4. Guaranteeing an increase in spread could also require consumption-smoothing forces that are strong enough, as in, e.g., Bocola, Bornstein, and DAVIS (2019), who study the role of subsistence consumption for borrowing behavior during debt crises. In our tractable model, with linear utility, we have disabled this channel, but we will revisit it in the quantitative analysis of our full model.

A Real Pricing Kernel. The pricing kernel is driven by the factor x_t , which follows an AR(1) process around a time-varying mean v_t , known at time t :

$$x_{t+1} = (1 - \rho)v_t + \rho x_t + \sigma_x \varepsilon_{t+1}. \quad (6)$$

v_t can be understood as a news shock, as it helps predict at time t near-term x_{t+k} dynamics. With probability p , $v_{t+1} = v_t$, and with complementary probability $1 - p$, v_{t+1} is drawn iid from a Normal distribution with mean μ_v and standard deviation σ_v , a renewal process:

$$v_{t+1} = \begin{cases} v_t, & \text{with probability } p \\ \text{iid } N(\mu_v, \sigma_v^2), & \text{otherwise.} \end{cases} \quad (7)$$

Finally, the real pricing kernel m_t is given by

$$-\log m_{t+1} = x_t + \frac{\lambda_m^2}{2} + \lambda_m \varepsilon_{t+1}, \quad (8)$$

where λ_m parameterizes the market price of risk. If $\sigma_v^2 = 0$, v would be constant across time and the model collapses into a standard one-factor kernel.

Inflation. Since estimating our pricing kernel will require us to use nominal yield curve data, even though our interest is in the underlying real kernel, we employ the following empirical specification for the gross Dollar inflation rate Π_t :

$$-\log \Pi_{t+1} = \mu_\pi + \iota_v v_t + \iota_x x_t + A_4(L)\eta_{t+1}. \quad (9)$$

The rate of inflation between periods $t + 1$ and t reflects a systematic response to real financial conditions (x_t, v_t) and an MA(4) shock process. We assume that η_{t+1} and ε_{t+1} are mutually independent, iid Standard Normal. We normalize $\iota_v = 1$ and $A_0 = 1$.

The Yield Curve in the Financial Center. As usual, given the real pricing kernel m_t and the gross inflation rate Π_t , zero coupon, nominal bond prices at all maturities n satisfy the recursion

$$q_t^{\$,n} = \mathbb{E}_t \left\{ \frac{m_{t+1}}{\Pi_{t+1}} q_{t+1}^{\$,n-1} \right\} \quad (10)$$

with initial condition $q_t^{\$,0} = 1$, and the nominal yield-to-maturity is $y_t^{\$,n} = -n^{-1} \log q_t^{\$,n}$.

Estimation. We aim to estimate the parameters of the system induced by (6–10) using quarterly data on US inflation and the US yield curve at 8 maturities between 1 and 120 quarters, $\{\Pi_t, q_t^{\$,1}, \dots, q_t^{\$,120}\}_{t=0}^{T-1}$. x_t and v_t are treated as latent. The expectation in (10) is conditional on the filtration $\mathcal{F}_t = \{x_t, v_t, \Pi_t, \{\eta_s\}_{s=t-3}^t\}$.

Unlike standard exponential affine models, where the yield curve can be expressed in closed form as a function of the factors, e.g., Ang and Piazzesi (2003), the presence of the jump term ν_t calls for a two-step procedure: first, we use the inflation rate and the 3mo yield to estimate the process for x_t and ν_t , in equations (6–7), then, using the yields at higher maturities, estimate the inflation equation (9) and the market price of risk parameter λ_m . Our estimation strategy is detailed in Appendix B.

	Estimate	95% CI
First Stage		
ρ_x	0.7395	[0.6123, 0.8183]
σ_x	0.0036	[0.0000, 0.0048]
σ_η	0.0045	[0.0000, 0.0524]
J	9	
p	0.9672	
μ_ν	$-8.2e^{-4}$	
σ_ν	0.0109	
Second Stage		
ι_x	-0.7425	
A_1	-0.4676	
A_2	-0.1445	
A_3	0.0369	
A_4	-0.0578	
λ_m	-0.2359	

Table 1: Stochastic Discount Factor Parameter Estimates

Table 1 compiles our parameter estimates. Our procedure detects $J = 9$ jumps in ν_t . The quarterly probability for $\nu_{t+1} = \nu_t$, p is roughly 0.97, implying a renewal of ν_t every seven and a half years. Figure 3 plots our estimates for ν_t and x_t in our sample, annualized, together with simulated 95% confidence bands for x_t and NBER-dated recession bars.

4 Quantitative Model

We propose a model with defaultable, long-term debt, augmented with stochastic pricing kernel of Section 3, and working capital domestic financial frictions following Neumeyer and Perri (2005). We argue that these elements are essential for replicating the impact of persistent and expected movements of rates in the international financial center on emerging markets’ production and spreads. Our model consists of international lenders, with the real pricing kernel of Section 3, the emerging market sovereign, and a domestic private sector. In turn, the private sector is composed of a representative household, a representative firm, and a representative financial intermediary.

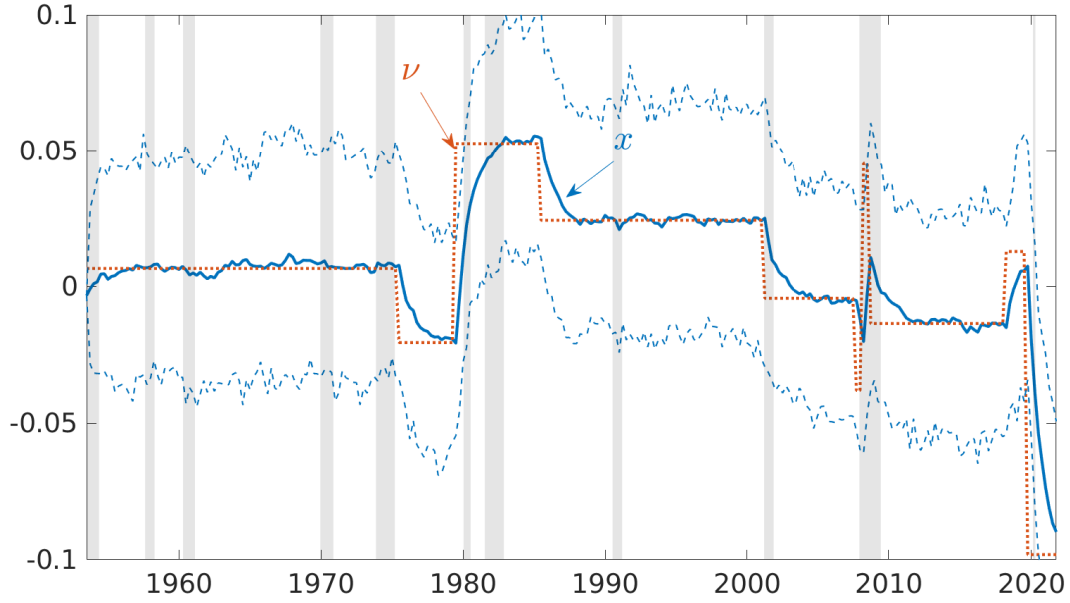


Figure 3: Estimates of ν_t and x_t , with 95% Simulated Confidence Bands (Annualized)

4.1 The Domestic Private Sector

Household. The representative household supplies labor competitively and receives the profits of domestic firms and financial intermediaries, as well as a lump-sum tax or transfer from the fiscal authority. We assume that it can trade a domestic bond which, in equilibrium, is in zero net supply. Its problem is given by

$$\max_{\{l_t, b_{t+1}^h\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \quad \text{s.t.} \quad c_t = w_t l_t + \Pi_t + \Pi_t^f + T_t - b_t^h + \frac{1}{1+i_t} b_{t+1}^h \quad (11)$$

where c_t is domestic consumption, w_t is the wage rate, l_t is labor supply, Π_t are the profits of domestic final goods producers, Π_t^f are the profits of financial intermediaries, and T_t is the fiscal authority's lump-sum transfer or tax. i_t is the domestic interest rate.

The household's behavior is summarized by its two first-order conditions, for labor supply,

$$u_\ell(c_t, l_t) + u_c(c_t, l_t) w_t = 0, \quad (12)$$

and asset holding, respectively,

$$u_c(c_t, l_t) = \beta(1+i_t) \mathbb{E}_t u_c(c_{t+1}, l_{t+1}). \quad (13)$$

In equilibrium, $b_t^h = b_{t+1}^h = 0$, and i_t is our measure of the domestic cost of funds, the rate relevant for financial intermediaries owned by the household.

Domestic Producer. Competitive domestic firms hire labor to produce final goods, using a decreasing return technology. A fraction θ of the wage bill must be financed with an *intra-period* working capital loan from the domestic financial intermediary.⁵ The profit maximization problem of the producer is thus

$$\Pi_t = \max_{\ell_t} \{A_t \ell_t^\alpha - [(1 - \theta) w_t \ell_t + \theta (1 + i_t) w_t \ell_t]\} \quad (14)$$

where i_t is the domestic interest rate carried by the intra-period working capital loan, ℓ_t is the resulting labor demand, and A_t is a productivity level common to all firms in the economy.

The resulting demand for labor is given by

$$\ell_t = \left(\frac{\alpha}{1 + \theta i_t} \cdot \frac{A_t}{w_t} \right)^{1/(1-\alpha)} \quad (15)$$

which implies that tighter domestic financial conditions, as captured by a higher interest rate for working capital loans depress production, at any given real wage.

We assume that the productivity level A_t is stochastic and endogenously lowered by the sovereign default decision, as described later. Net of any such reductions induced by sovereign default, the productivity term evolves exogenously, according to

$$\log A_{t+1} = \rho_A \log A_t + \sigma_A \varepsilon_{t+1} \quad (16)$$

and $\varepsilon_{t+1} \sim \mathcal{N}(0, 1)$ is standard Normal.

Domestic Financial Intermediary. The domestic financial intermediary extends working capital loans to producers and transfers the profits resulting from this activity to the household

$$\Pi_t^f = -a_t + (1 + i_t) a_t = i_t a_t, \quad (17)$$

where a_t is the working capital loan size. In equilibrium firms take loans of size $a_t = \theta w_t \ell_t$.

The rate for this intra-period loan is given by the domestic interest rate

$$i_t = \frac{u_c(c_t, \ell_t)}{\beta \mathbb{E}_t u_c(c_{t+1}, \ell_{t+1})} - 1, \quad (18)$$

which is the domestic cost of funding in this economy, measured by the expected marginal rate of intertemporal substitution of the household. This is consistent with common timing assumptions made in modeling intra-period loans, e.g., the loan is extended in the “morning” of period t while production and repayments are done in the “evening” of period t . We use this structure to combine the benefit from having the marginal rate of substitution of the representative household

5. A similar constraint is used by Mendoza and Yue (2012) to expose the import of intermediate goods to default risk, to endogenize the resource costs of default. Modeling such intra-period loans follows Fuerst (1992).

determine the domestic cost of borrowing, with the tractability of the intra-period loan, so that no additional state variables are required.

4.2 Sovereign Borrowing and Default

A benevolent fiscal authority can borrow abroad, lacking commitment over future repayment and borrowing decisions. It starts each period t with an initial debt level B_t , which, if it chooses not to default, it must service with a payment of κB_t , at which point it can issue or retire units of its long-term bond at price q_t , inducing next period's debt level B_{t+1} . The resulting proceeds from operating in international markets are transferred lump sum to the domestic household⁶, net of a constant public spending level \bar{G} ,

$$T_t = -\kappa B_t + q_t [B_{t+1} - (1 - \delta) B_t] - \bar{G}. \quad (19)$$

If the sovereign chooses to default, we encode this decision in a policy variable $d_t = 1$, no debt service payments are made and no new issuance or buyback of bonds are possible, resulting in $T_t = -\bar{G}$. If instead the sovereign services its debt, we write $d_t = 0$.

We assume that, following a decision to default, the sovereign spends a random length of time excluded from international financial markets. With a constant probability χ the sovereign resumes market access without outstanding debt. Furthermore, a state of default carries a direct productivity reduction for domestic producers, that is $A_t^d \leq A_t$, reflecting in reduced-form any financial or trade disruptions caused by the sovereign's default choice.⁷

International Lenders. Competitive international lenders price the sovereign's bonds, given their real pricing kernel m_{t+1} ,

$$q_t = \mathbb{E}_t \{ m_{t+1} (1 - d_{t+1}) [\kappa + (1 - \delta) q_{t+1}] \}. \quad (20)$$

The yield-to-maturity spread of the sovereign bond is given by $\kappa/q_t - \kappa/q_t^{\text{rf}}$ where q_t^{rf} is the price of the default-risk-free bond trade in the financial center, with the same maturity structure with the sovereign's bond, satisfying

$$q_t^{\text{rf}} = \mathbb{E}_t \left\{ m_{t+1} \left[\kappa + (1 - \delta) q_{t+1}^{\text{rf}} \right] \right\}. \quad (21)$$

6. We consider a setting with centralized borrowing, as standard in the quantitative sovereign default literature, by assuming that all external lending or borrowing is done by the government, some of which on behalf of the private sector. Possible alternatives, which we do not pursue here, are combinations of (de)centralized borrowing and (de)centralized default, with the caveat that, with flexible enough capital controls policy instruments, the government might still be able to induce the outcomes studied here.

7. Mendoza and Yue (2012) endogenize such a productivity cost of default through an explicit model of imported intermediate goods subject to working capital loans. Arellano, Bai, and Bocola (2017) and Sosa-Padilla (2018) study the consequences of default for domestic activity through its impact on domestic banks' balance sheet.

4.3 A Recursive Formulation

We consolidate our model and provide a recursive formulation, using exogenous state variables $s = \langle x, v, A \rangle$ and the endogenous state variable B . Implicitly, whether the sovereign has or lacks market access is also recorded as a state variable but we choose to express this dependency by writing down separate value and policy functions for a country in default. As standard in the study of equilibrium default, we restrict attention to Markov Perfect Equilibria.

Domestic Outcomes. In state $\langle s, B \rangle$, if the sovereign chooses B' as next period's debt level, domestic outcomes are summarized by policy functions $\mathcal{Y}(s, B, B')$ for GDP, $\mathcal{C}(s, B, B')$ for consumption, $\mathcal{L}(s, B, B')$ for employment, $\mathcal{I}(s, B, B')$ for the domestic interest rate, the wage rate $\mathcal{W}(s, B, B')$, and the consolidated profits of firms and financial intermediaries $\mathcal{P}(s, B, B')$. These satisfy, suppressing state dependency, the labor supply condition

$$u_\ell(\mathcal{C}, \mathcal{L}) + u_c(\mathcal{C}, \mathcal{L}) \mathcal{W} = 0 \quad (22)$$

the labor demand condition

$$\mathcal{L} = \left(\frac{\alpha}{1 + \theta \mathcal{I}} \cdot \frac{A}{\mathcal{W}} \right)^{1/(1-\alpha)} \quad (23)$$

the household's budget constraint, once consolidate with the fiscal balance,

$$\mathcal{C} = \mathcal{W}\mathcal{L} + \mathcal{P} - \kappa B + q(s, B') [B' - (1 - \delta)B] - \bar{G} \quad (24)$$

a profit condition for \mathcal{P} consistent with the problems in Section 4.1, and the domestic interest rate condition

$$u_c(\mathcal{C}, \mathcal{L}) = \beta (1 + \mathcal{I}) H(s, B'), \quad (25)$$

for any state and choice $\langle s, B, B' \rangle$, where q is the bond price schedule and H is expected future marginal utility of consumption, both to be defined shortly. GDP is $\mathcal{Y} = A\mathcal{L}^\alpha = \mathcal{W}\mathcal{L} + \mathcal{P}$.

Domestic Outcomes in Default. Analogously, domestic outcomes in default are characterized by functions of $\langle s \rangle$ alone, as there is no borrowing choice, e.g., $\mathcal{Y}^d(s), \mathcal{C}^d(s), \mathcal{L}^d(s), \dots$, and an expected marginal utility of consumption while in default, $H^d(s)$.

Forward-looking Functions. The bond price schedule is given by

$$q(s, B') = \mathbb{E}_{s'|s} \{ m(s, s') (1 - \mathcal{D}(s', B')) [\kappa + (1 - \delta)q(s', \mathcal{B}(s', B'))] \} \quad (26)$$

where $\mathcal{B}(s, B)$ and $\mathcal{D}(s, B)$ are the sovereign's equilibrium issuance and default policies function, to be described shortly.

The expected marginal utility functions, with market access and in default, can be expressed

parsimoniously, by introducing an auxiliary J function

$$J(s, B) = [1 - \mathcal{D}(s, B)] u_c(\mathcal{C}(s, B, \mathcal{B}(s, B)), \mathcal{L}(s, B, \mathcal{B}(s, B))) + \mathcal{D}(s, B) u_c(\mathcal{C}^d(s), \mathcal{L}^d(s)), \quad (27)$$

as

$$H(s, B') = \mathbb{E}_{s'|s} J(s', B') \quad (28)$$

and

$$H^d(s) = \mathbb{E}_{s'|s} \left\{ \chi J(s', 0) + (1 - \chi) u_c(\mathcal{C}^d(s'), \mathcal{L}^d(s')) \right\}. \quad (29)$$

The expected marginal utilities here account for the risk of default in certain future states next period, and the associated low levels of consumption, as in Arellano, Bai, and Mihalache (2020). Equilibrium default shapes the domestic rate i in key states of the world, through the H function.

Sovereign. The sovereign chooses whether to default,

$$V(s, B) = \max_{d \in \{0,1\}} \left\{ d V^d(s) + (1 - d) V^r(s, B) \right\} \quad (30)$$

with its choice encoded in the policy function $\mathcal{D}(s, B)$, and how much debt to carry into next period, conditional on not defaulting,

$$V^r(s, B) = \max_{B'} \left\{ u(\mathcal{C}(s, B, B'), \mathcal{L}(s, B, B')) + \beta \mathbb{E}_{s'|s} V(s', B') \right\} \quad (31)$$

with solution encoded in the policy function $\mathcal{B}(s, B)$. The value in default satisfies

$$V^d(s) = u(\mathcal{C}^d(s), \mathcal{L}^d(s)) + \beta \mathbb{E}_{s'|s} \left\{ \chi V(s', 0) + (1 - \chi) V^d(s') \right\} \quad (32)$$

so that the sovereign has no choice variables but understands that domestic outcomes are consistent with labor and working capital markets clearing.

Definition of Equilibrium. A *Markov Perfect Equilibrium* consists of

- Value functions for the sovereign, V, V^r, V^d
- Policy functions for the sovereign, \mathcal{D}, \mathcal{B}
- Domestic outcome functions with market access, $\mathcal{C}, \mathcal{L}, \dots$
- Domestic outcome functions in default, $\mathcal{C}^d, \mathcal{L}^d, \dots$
- The bond price schedules q ,
- Expected marginal utility of consumption functions, H, H^d

such that

- The sovereign’s policy functions solve the maximization problems of the value function, given domestic outcomes
- Bond prices are consistent with the sovereign’s policy functions, lenders break even
- The expected marginal utility functions are consistent with the sovereign’s policies and domestic outcomes
- Domestic outcomes are consistent with market clearing for labor and working capital, in default and with market access, respectively.

5 Quantitative Analysis

We calibrate our model and compare its quantitative properties to the data and to a version without domestic production subject to financial frictions, corresponding to the case of $\theta = 0$.

5.1 Calibration and Quantitative Properties

We assume household preferences are CRRA and GHH, following Greenwood, Hercowitz, and Huffman (1988),

$$u(c, \ell) = \frac{\left(c - \psi \frac{\ell^{1+\mu}}{1+\mu}\right)^{1-\sigma} - 1}{1-\sigma}. \quad (33)$$

The functional form for the cost of default in terms of productivity follows Chatterjee and Eyigungor (2012),

$$h(A) = A - \max\{0, \lambda_0 A + \lambda_1 A^2\} \leq A \quad (34)$$

and we focus on the concave case, $\lambda_0 \leq 0 \leq \lambda_1$.

Table 2 summarizes parameter values for our quarterly calibration. There are three groups of parameters. First, some parameters are set to conventional values in the literature, including the coefficient of risk aversion σ , the discounting factor β , and a Frisch labor supply elasticity of 1, together with labor’s share or return to scale α , the maturity of the debt δ and the market return probability χ . A second group of parameters are those that discipline the lenders’ pricing kernel, the processes for x and ν . We set these based on our estimated in Section 3 directly. A final set of parameters are used to match key moments in the data: the default penalty parameters λ_0 and λ_1 , the working capital requirement θ , and the process of productivity, ρ_A and σ_A .

We compute this model with defaultable long-term debt using *discrete choice methods*, as discussed in Appendix D. Recent applications of these methods for default models and their rationale can be found in, among others, Mihalache (2020), Dvorkin et al. (2020), Dvorkin et al. (2021), or Wolf and Zessner-Spitzenberg (2021), including comparisons with prior methods based on other iid perturbations, such as Chatterjee and Eyigungor (2012). This method requires the introduction of two additional relaxation parameters, η_D for the default decision and η_B for the borrowing decision.

	Value	Comment
σ	2.0	CRRA
β	0.99	Discounting
ψ	1.0	Normalization, mean ℓ
μ	1.0	Inverse of Frisch elasticity
α	0.67	Returns to scale
θ	1.5	Working capital constraint
\bar{G}	0.0	Public spending
δ	0.05	5 year debt Macaulay duration
κ	$\delta + \mu_v$	Normalization
λ_0	-0.24	Penalty, linear
λ_1	+0.27	Penalty, quadratic
χ	1/8	Market return probability
ρ_x	0.74	Autocorrelation of pricing kernel factor
σ_x	0.0036	Volatility of factor
μ_v	$-8.3e^{-4}$	Average factor level
σ_v	0.011	Volatility of factor trend shocks
p	0.9672	Probability of renewal
λ_m	-0.236	Market price of risk
ρ_A	0.95	Autocorrelation of productivity
σ_A	0.005	Volatility of productivity shock
η_D	$1e^{-6}$	Default taste shock
η_B	$5e^{-5}$	Borrowing taste shock

Table 2: Parameter Values

	Data	Model
Mean		
Spread		1.96
Debt to GDP		12.3
Standard Deviations		
Spread		1.37
GDP		3.91
Consumption		4.87
Domestic Rate		3.51
Correlations		
Spread and GDP		-24.1
Trade Balance to GDP and GDP		-67.4

Table 3: Moments

5.2 Impulse Response Functions

We produce stochastic impulse response functions for our model following the method of Koop, Pesaran, and Potter (1996). This approach is particularly well suited for models of sovereign default due to the highly nonlinear nature of policy functions around the dichotomous default decision.⁸ We construct these impulse response panels by simulating a wide and long panel of uncorrelated countries, such that after a long enough time the cross-sectional distribution converges to the ergodic distribution of the model. We then shock at time $t = 0$ all economies by the same amount and plot average dynamics over time. We exclude the simulated economies that are in a state of default, and confirm that results are similar if we were to include them.

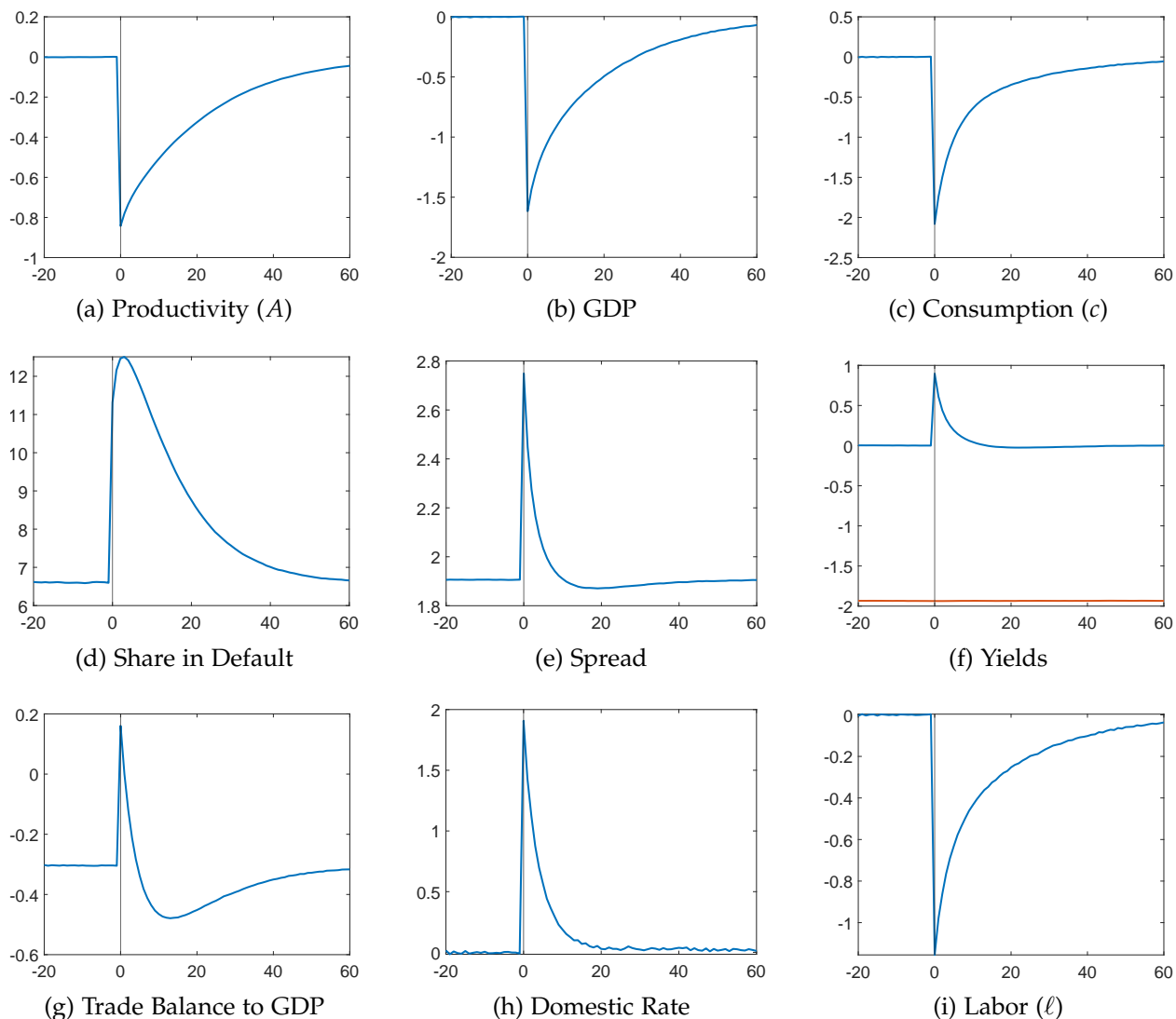


Figure 4: Impulse Response Functions: Productivity Shock (A)

Figure 4 plots the responses to a 0.8% drop in productivity. It leads to a 1.6% drop in GDP

8. For other applications of this approach to impulse response functions to sovereign default models see, among others, Arellano, Bai, and Mihalache (2018) and Dvorkin et al. (2020).

and about a 2% drop in consumption. The share of economies (in the cross-section of our simulated panel) in a state of default goes from 6.75% to under 13%, so about 6% of economies immediately default. Conditional on not defaulting, the spread spikes by about 0.6%. The trade balance exhibits the standard Current Account reversal pattern, as capital flows reverse. Domestic financial conditions are tight, with a 2% increase in the domestic rate, reflecting low consumption prospects in the future, due to default risk and persistently low productivity. Employment drops by 1.1%, due to both low productivity and high working capital loan rates. This patterns are standard for sovereign default models with GHH preferences, except for the additional amplification coming from the working capital requirement.

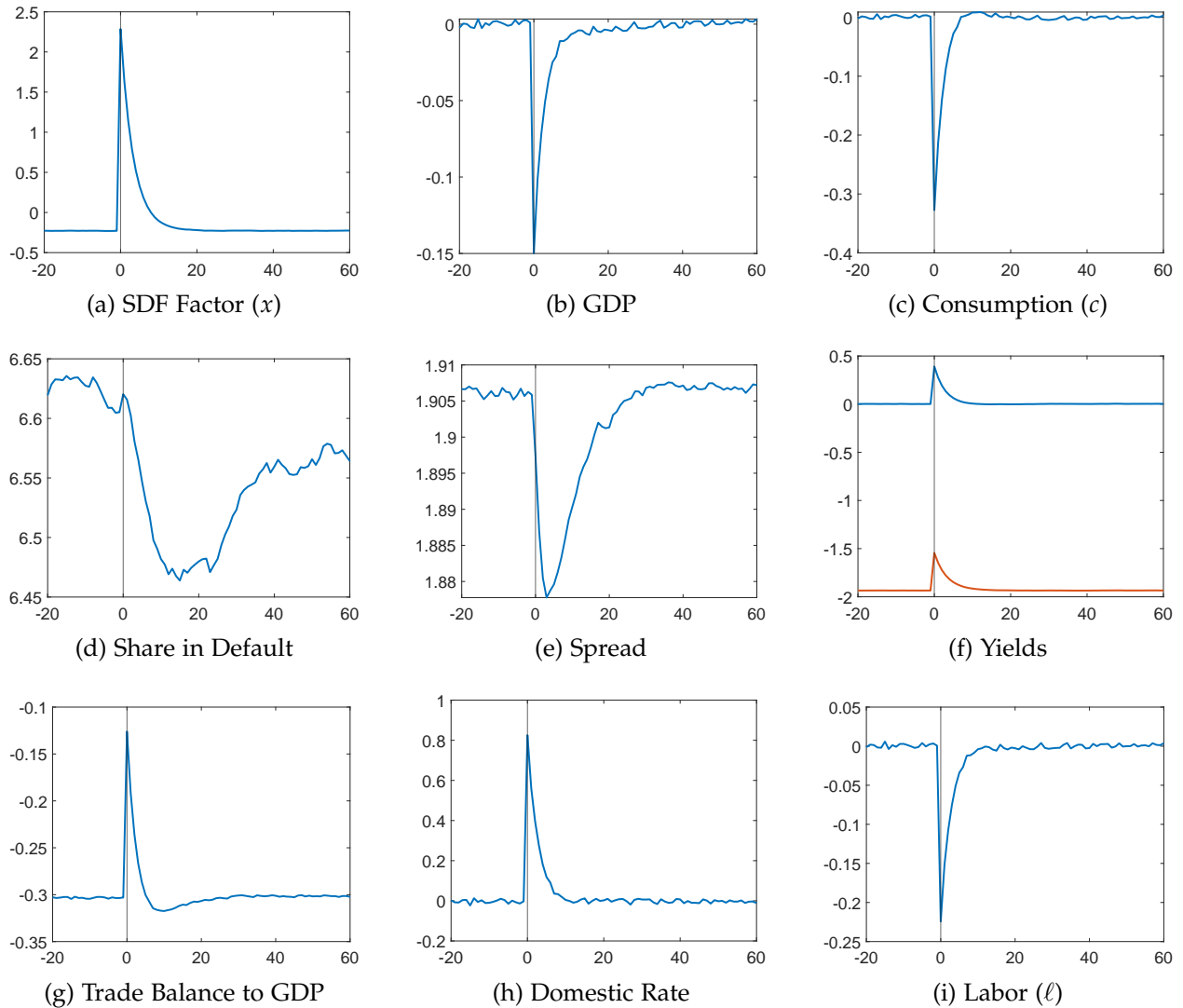


Figure 5: Impulse Response Functions: SDF Factor (x)

Figure 5 displays the case of a shock to the pricing kernel factor, x . We increase x by about 2.3%, which leads to a 0.5% increase in the yield of the risk-free long-term bond (the orange line in panel (f)) and drops in GDP and consumption of 0.15% and 0.3%, respectively. The drop in GDP

is due to the 0.21% drop in employment, induced by tight financial conditions in the domestic market: i increases by about 0.8%. Spreads fall and the share of economies in default shrinks over time, as sovereigns aggressively delever.

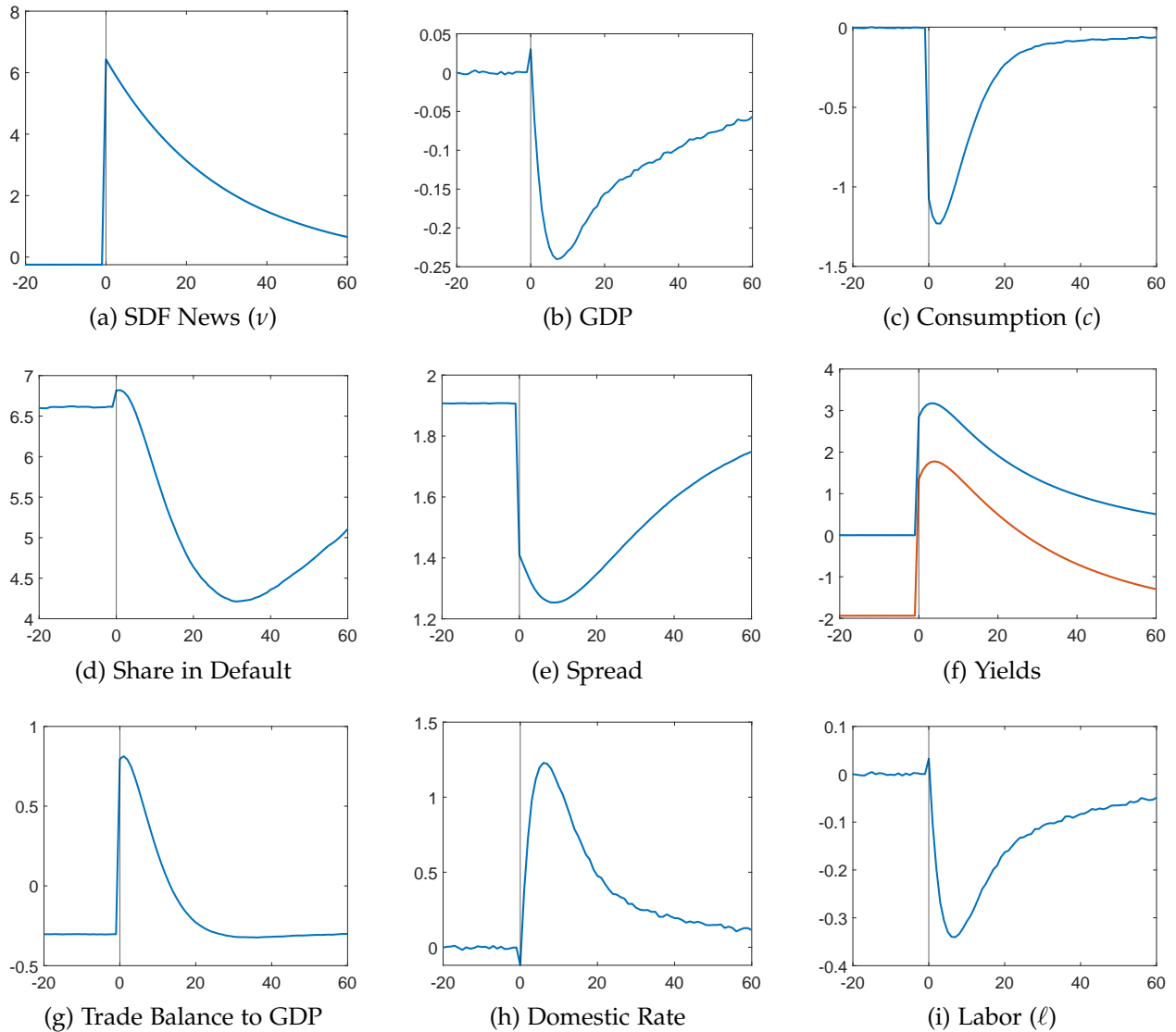


Figure 6: Impulse Response Functions: SDF News (ν)

The responses to a 6% news shock ν , the persistent conditional mean of the one-step ahead forecast of the factor, are plotted in Figure 6. The yield of the long-term risk-free bond immediately jump by about 1.5% and continues to increase for several periods, as x tends on average to converge to the new, higher ν level. Eventually, as ν resets with probability $1 - p$, yields return to their initial, average levels. Domestic outcomes exhibit hump-shape patterns. The domestic rate increases smoothly and peaks about 8-10 quarters after the initial news. Employment and GDP follow the same timing as the domestic rate. In contrast, consumption falls immediately on impact, because of the sovereign's prompt reduction in borrowing. The economy exhibits are sharp and prolonged

Current Account reversal, and spreads fall by about 0.6% and remain below average for several years.

In summary, tight conditions in the financial center are robustly recessionary, and induce higher domestic rates in the emerging market as well. Surprisingly, both in the face of highly transitory (x) and expected and persistent (v) shocks, spreads fall. This is because the sovereign, while generally willing to borrow into a tight bond price schedule, chooses to delever and cut B' aggressively.

5.3 Domestic Policies

To better showcase the impact of sovereign borrowing on domestic outcomes, in Figure 7, we fix all shocks to their mean levels and the debt level B close to its unconditional average, and vary the choice of debt position for next period (B'). We mark with a solid black line the current B level and with a dashed black line the equilibrium choice for B' . The three panels show, in turn, consumption, the domestic rate, and employment, across B' levels.

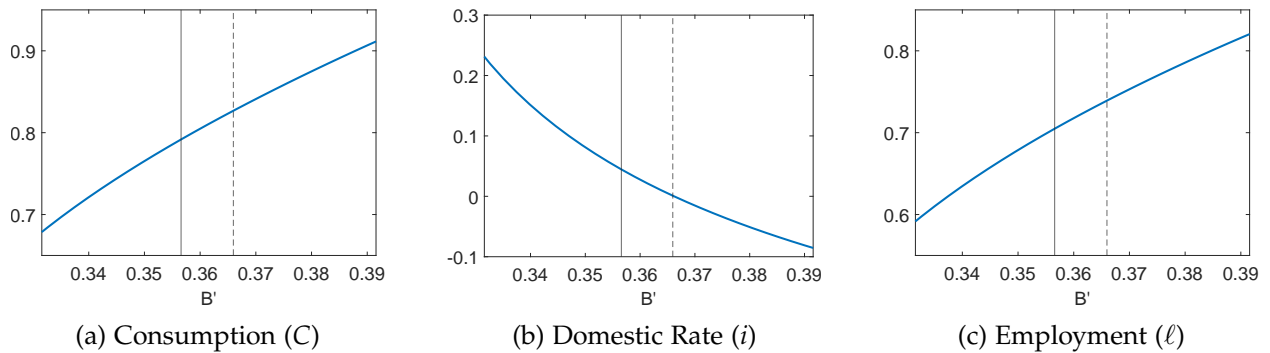


Figure 7: Domestic Outcomes, the Role of Borrowing

As the sovereign borrows more, and since we're in a region of the state state where the bond price q falls only modestly with B' , consumption increases, domestic financial conditions are looser, and employment and production expand. As a result, capital inflows are expansionary in this setting. Note that in all standard sovereign default models, capital inflows are associated with higher endowment or income levels, as favorable fundamentals loosen the bond price schedule and the sovereign endogenously borrows more. Instead, here, counterfactually higher borrowing is expansionary, keeping shocks fixed.

6 Conclusion

[TODO]

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A Proofs

A.1 Proposition 1

From the Equation 5 in the main text, the first-order condition under linear utility is

$$1 - h[v^*(b')] b' = \beta (1 + r^{rf}).$$

The derivative of $h(v^*(b')) b'$ with respect to b' is

$$h'(v^*(b')) v^{*'}(b') b' + h(v^*(b')) \quad (35)$$

The default utility loss $v^*(b') = V^d - \bar{v} + b'$ increases with b' . The first derivative of hazard function is:

$$\begin{aligned} h'(v) &= \frac{f'(v)(1-F(v)) + f^2(v)}{(1-F(v))^2} \\ &= \frac{f'(v)}{1-F(v)} + \left(\frac{f(v)}{1-F(v)} \right)^2 \end{aligned} \quad (36)$$

If v follows a distribution with first derivative of density function is great than 0, then the first derivative of hazard function is positive. It applies to uniform distribution and normal distribution. For standard normal distribution,

$$\begin{aligned} h'(v) &= \frac{-vf(v)}{1-F(v)} + \left(\frac{f(v)}{1-F(v)} \right)^2 \\ &= -vh(v) + h^2(v) \\ &= h(v)(h(v) - v) \end{aligned} \quad (37)$$

$h'(v) > 0$ by property of inverse of Mills' ratio⁹. For a more generalized normal distribution with $\sigma_v v \sim \mathcal{N}(0, \sigma_v^2)$, $h'(v) > 0$ still holds.

Thus, a one-time positive shock to r^{rf} will make b' goes down. Default probability/risk $\Phi[v^*(b')]$ decreases with b' and spreads $= \frac{1+r^{rf}}{1-\Phi[v^*(b')]} - r^{rf}$ decreases with b' .

A.2 Proposition 2

[TODO]

B Estimation Procedure for Financial Center SDF

We have quarterly observations of the random vector $(\{-\Pi_{t+1}, q_t^1, \dots, q_t^{120}\}_{t=0}^{T-1})$ series (inflation and 3mo to 30y yields) to estimate the unknown parameters of the model. Since $\{x_t, v_t\}$ are latent,

9. $h'(v) > 0$ is obvious for $v < 0$. For $v \geq 0$, $h'(v) > 0$ according to property of inverse of Mills' ratio. Baricz (2008)

we proceed in two steps.

Step 1. Taking the log of q_t^1 from equation (10), we get

$$\log(q_t^1) = -(1 - \iota_x)x_t + \mu_\pi + \nu_t + 0.5\sigma_\eta^2 + \sum_{i=0}^3 A_{i+1}\eta_{t-i}, \quad (38)$$

which implies that

$$\begin{aligned} \tilde{x}_t &= -\pi_{t+1} - \log(q_t^1) = \mu_\pi + \nu_t + \iota_x x_t + A_4(L)\eta_{t+1} + (1 - \iota_x)x_t - \mu_\pi - \nu_t - 0.5\sigma_\eta^2 - \sum_{i=0}^3 A_{i+1}\eta_{t-i} \\ &= -0.5\sigma_\eta^2 + x_t + \eta_{t+1}. \end{aligned} \quad (39)$$

Since η is a Gaussian white noise, \tilde{x}_t is a mismeasured version of x_t . Moreover, by plugging in (9), we get

$$\begin{aligned} \tilde{x}_t &= -0.5\sigma_\eta^2 + x_t + \eta_{t+1} \\ &= -0.5\sigma_\eta^2 + (1 - \rho)v_{t-1} + \rho x_{t-1} + \sigma\varepsilon_t + \eta_{t+1} \\ &= -0.5(1 - \rho)\sigma_\eta^2 + (1 - \rho)v_{t-1} + \rho\tilde{x}_{t-1} + \sigma\varepsilon_t + \eta_{t+1} - \rho\eta_t \\ &= -0.5(1 - \rho)\sigma_\eta^2 + (1 - \rho)v_{t-1} + \rho\tilde{x}_{t-1} + u_t - \rho u_{t-1}, \end{aligned} \quad (40)$$

where u_t is a Gaussian white noise with mean 0 and variance equal to $\sigma_\eta^2 + \frac{\sigma^2}{1+\rho^2}$. This has two implications. First, the nonstationarity in the mean of x_t is reflected in the nonstationarity in the mean of \tilde{x}_t . Second, conditional on v_{t-1} , \tilde{x}_t follows an ARMA(1, 1) process (Staudenmayer and Buonaccorsi 2005).

Let us rewrite equation (40) as follows

$$\begin{aligned} \tilde{x}_t &= -0.5(1 - \rho)\sigma_\eta^2 + (1 - \rho)v_{t-1} + \rho\tilde{x}_{t-1} + u_t - \rho u_{t-1} \\ &= -0.5(1 - \rho)\sigma_\eta^2 + (1 - \rho) \sum_{j=0}^J \beta_j \mathbb{1}(t \geq T_j) + \rho\tilde{x}_{t-1} + u_t - \rho u_{t-1}, \end{aligned} \quad (41)$$

where $\{T_j, j = 0, \dots, J\}$ represents jump points (the break points of v , with $T_0 = 0$), and the β_j s measure their size. Equation (41) is an ARMA(1, 1) process with multiple structural breaks in the mean (Bai and Perron 1998). The number of breaks, J , and the locations of the breaks, $\{T_j, j = 1, \dots, J\}$, are unknown and need to be estimated. If the number and locations of the breaks were known, and since u_t follows a normal distribution, the model above can be efficiently estimated by maximum likelihood. However, since J and $\{T_j, j = 1, \dots, J\}$ are unknown in practice, they need to be estimated along with the other parameters. To estimate J and $\{T_j, j = 1, \dots, J\}$, we follow the binary segmentation approach suggested by Bai and Perron (1998). We first consider the estimation of the model with a single break, and we place that break at the point where the Sum of Squared Residuals (SSR) of model (41) reaches its minimum. Since \tilde{x}_{t-1} is an endogenous

regressor (because of its correlation with u_{t-1}), we compute the SSR using an instrumental variable approach, where \tilde{x}_{t-2} is used as a valid instrument. Then we split the sample into two subsamples, find the break point in each of the two subsamples, and then take as second breakpoint the one in which the SSR of the model with two breaks is minimized. We iterate this approach for any number of breaks, and we choose \hat{j} as the point in which the Schwartz Criterion is minimized (Yao 1988). Hence, from the estimation of (41), we directly obtain $(\hat{\rho}, \hat{\mu}_v, \hat{\sigma}_v, \hat{\sigma}, \hat{\sigma}_\eta, \hat{j}, \hat{T}_1, \dots, \hat{T}_j)$. The probability that v_t does not jump, p , is estimated as

$$\hat{p} = 1 - \frac{\hat{j}}{T-1},$$

and

$$\hat{\mu}_v = \frac{1}{\hat{j}} \sum_{j=0}^{\hat{j}} \sum_{l=0}^j \hat{\beta}_l, \quad \hat{\sigma}_v = \sqrt{\frac{1}{\hat{j}-1} \sum_{j=0}^{\hat{j}} \left(\sum_{l=0}^j \hat{\beta}_l \right)^2},$$

the mean and the variance of the estimated size of the jumps. When there are no jumps $\hat{\mu}_v$ corresponds to the estimator of the overall mean of \tilde{x}_t (which is a stationary process), and $\hat{\sigma}_v = 0$.

Estimated parameters from the model in equation (41) are reported in Table 1 along with their 95% profile likelihood confidence intervals. Figure 3 gives instead the dynamics of the latent factor (x_t , solid blue line), with the 95% simulated confidence interval, and the estimated conditional mean v_t , given (x_{t-1}, v_{t-1}) .

Step 2. For each $t = 1, \dots, T$, let $Y_{t+1} = (q_t^1, \dots, q_t^{120})'$, a 1×8 vector of observations. \mathbf{Y} is the $T \times 8$ matrix which is obtained by stacking them. To estimate the remaining parameters, $(\iota_x, A_1, \dots, A_4, \lambda)$, we use a Simulated Method of Moment (SMM) approach. Once the other parameters have been fixed, we can sample from the joint distribution of (ε, η) , to generate synthetic observations of Y_{t+1} , that we denote Y_{t+1}^* . We generate $\{Y^{*,r}, r = 1, \dots, R\}$, and we choose $(\hat{\iota}_x, \hat{A}_1, \dots, \hat{A}_4, \hat{\lambda})$, such that

$$(\hat{\iota}_x, \hat{A}_1, \dots, \hat{A}_4, \hat{\lambda}) = \arg \min_{(\iota_x, A_1, \dots, A_4, \lambda)} \frac{1}{R} \sum_{r=1}^R \|\mathbf{Y}^{*,r} - \mathbf{Y}\|_2^2, \quad (42)$$

where, for a matrix B , $\|\cdot\|_2$ denotes the spectral norm of B (i.e., the square-root of the largest eigenvalue of $B'B$). Loosely speaking, we choose the remaining parameters to minimize the average distance between observed and simulated values.

The number of simulated samples R is chosen in such a way that $\sqrt{T}/R = o(1)$ to guarantee that the bias generated by simulations goes to zero faster than \sqrt{T} (Gouriéroux and Monfort 1997). Specifically, we take $R = 2\sqrt{T} \log(T)$. Estimates of these parameters from the minimization of (42) are given in the second half of Table 1.

C Data Sources

[TODO]

D Computation with Taste Shocks

We compute our model with defaultable, long-term debt using discrete choice methods, following Mihalache (2020) and Dvorkin et al. (2021). We introduce Extreme Value Type I shocks for the choices over B' and d , so that ex-ante there is a choice probability function over B' levels, as well as an interior default probability for any ex-post state. These taste shocks facilitate convergence in models with defaultable long-term debt and smooth policies and values. The appendix of Arellano, Bai, and Mihalache (2020) provides additional discussion for these methods.

The Domestic Economy. With GHH preferences, for any arbitrary B' choice, the domestic allocation is summarized by

$$\begin{cases} \mathcal{L}(s, B, B') = \left[\frac{\alpha}{\psi} \cdot \frac{A}{1+\theta \mathcal{I}(s, B, B')} \right]^{1/(1-\alpha+\mu)} \\ \mathcal{C}(s, B, B') = A \mathcal{L}(s, B, B')^\alpha + T(s, B, B') \\ \left[\mathcal{C}(s, B, B') - \psi \frac{\mathcal{L}(s, B, B')^{1+\mu}}{1+\mu} \right]^{-\sigma} = \beta (1 + \mathcal{I}(s, B, B')) H(s, B') \end{cases} \quad (43)$$

which, for given T and H functions, can be reduced one equation in one unknown, $\mathcal{I}(s, B, B')$, which we solve with standard one-dimensional root-finding methods. Then, given $\mathcal{I}(s, B, B')$, we construct $\mathcal{C}(s, B, B')$ and $\mathcal{L}(s, B, B')$. The fiscal transfer T is given by $T(s, B, B') = -\kappa B + q(s, B') [B' - (1 - \delta)B] - \bar{G}$, which reflects the bond price schedule q .

Analogously, in default domestic outcomes satisfy

$$\begin{cases} \mathcal{L}^d(s) = \left[\frac{\alpha}{\psi} \cdot \frac{h(A)}{1+\theta \mathcal{I}^d(s)} \right]^{1/(1-\alpha+\mu)} \\ \mathcal{C}^d(s) = h(A) (\mathcal{L}^d(s))^\alpha - \bar{G} \\ \left[\mathcal{C}^d(s) - \psi \frac{(\mathcal{L}^d(s))^{1+\mu}}{1+\mu} \right]^{-\sigma} = \beta (1 + \mathcal{I}^d(s)) H^d(s). \end{cases} \quad (44)$$

Default Choice. With taste shocks, the default decision becomes

$$V(s, B) = \eta_D \log \left\{ \exp \left[\frac{V^d(s)}{\eta_D} \right] + \exp \left[\frac{V^r(s, B)}{\eta_D} \right] \right\} - \eta_D \cdot \text{em} \quad (45)$$

where em is the Euler-Mascheroni constant, and the default probability after observing the state s but before observing the taste shocks is

$$d(s, B) = \frac{1}{1 + \exp \left[\frac{V^r(s, B) - V^d(s)}{\eta_D} \right]}. \quad (46)$$

The parameter η_D controls the amount of “noise” in the default decision.

Sovereign Borrowing Choice. For each possible new debt level B' , denote the associated value

$$W(s, B, B') = u \left(\mathcal{C}(s, B, B'), \mathcal{L}(s, B, B') \right) + \beta \mathbb{E}_{s'|s} V(s', B') \quad (47)$$

and set $W(s, B, B') = -\infty$ for all infeasible B' values. Then,

$$V^r(s, B) = \eta_B \log \left\{ \sum_{B'} \exp \left[\frac{W(s, B, B')}{\eta_B} \right] \right\} - \eta_B \cdot \text{em} \quad (48)$$

and the probability of choosing any particular $B' = z$ after observing s is given by

$$\Pr(B' = z | s, B) = \frac{\exp \left[\frac{W(s, B, z)}{\eta_B} \right]}{\sum_j \exp \left[\frac{W(s, B, j)}{\eta_B} \right]}. \quad (49)$$

The Value of Default. The value of default satisfies

$$V^d(s) = u \left(\mathcal{C}^d(s), \mathcal{L}^d(s) \right) + \beta \mathbb{E}_{s'|s} \left\{ (1 - \chi) V^d(s') + \chi V(s', 0) \right\}.$$

Bond Prices. The choice probabilities for default and borrowing are reflected in the lenders' pricing of the debt:

$$q(s, B') = \mathbb{E}_{s'|s} m(s, s') (1 - d(s', B')) \left[\kappa + (1 - \delta) \sum_{B''} \Pr(B'' | s', B') q(s', B'') \right]. \quad (50)$$

Expected Marginal Utility. The functions expressing expected marginal utility reflect taste shocks as well. We define

$$\begin{aligned} J(s, B) &= [1 - d(s, B)] \sum_{B'} \Pr(B' | s, B) \left[\mathcal{C}(s, B, B') - \psi \frac{\mathcal{L}(s, B, B')^{1+\mu}}{1 + \mu} \right]^{-\sigma} \\ &\quad + d(s, B) \left[\mathcal{C}^d(s) - \psi \frac{(\mathcal{L}^d(s))^{1+\mu}}{1 + \mu} \right]^{-\sigma} \end{aligned} \quad (51)$$

and use it to construct

$$H(s, B') = \mathbb{E}_{s'|s} J(s', B') \quad (52)$$

and

$$H^d(s) = \mathbb{E}_{s'|s} \left\{ \chi J(s', 0) + (1 - \chi) \left[\mathcal{C}^d(s) - \psi \frac{(\mathcal{L}^d(s))^{1+\mu}}{1 + \mu} \right]^{-\sigma} \right\}. \quad (53)$$

Algorithm. [TODO]

1. ...

E Discrete Approximation of the Pricing Kernel

The original pricing kernel:

$$x_{t+1} = (1 - \rho)v_t + \rho_x x_t + \sigma_x \varepsilon_{t+1}. \quad (54)$$

$$v_{t+1} = \begin{cases} v_t & \text{w.p. } p \\ \text{iid } N(\mu_v, \sigma_v^2) & \text{otherwise.} \end{cases} \quad (55)$$

$$-\log m_{t+1} = x_t + \lambda_m^2/2 + \lambda_m \varepsilon_{t+1}, \quad (56)$$

Discretize v using grid \vec{v} and transition matrix Π_v .

Grid \vec{x} for x and transition probabilities $\Pi_{x|v} = \Pr(x'|x, v)$ for x , conditional on v .

Transition $\Pi_{x,v} = \Pr(x', v'|x, v) = \Pi_{x|v} \Pi_v$.

$$-\log m(x, v, x') = \frac{\lambda_m^2}{2} + x + \frac{\lambda_m}{\sigma_x} (x' - (1 - \rho_x)v - \rho_x x)$$