The Maturity and Payment Schedule of Sovereign Debt*

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June 9, 2016

Abstract

This paper addresses a long-standing puzzle about the maturity of newly issued emerging market debt, over the cycle, by studying the timing and relative size of scheduled payments. Using *Bloomberg* bond data for eleven emerging economies, we document that countries react to crises by issuing debt with shortened maturity but more back-loaded payment schedules, than on average. To account for these patterns, we develop a sovereign default model with an endogenous choice of both maturity and payment schedule. During recessions, a country prefers to borrow using a schedule that is more back-loaded—delaying relatively larger payments to dates closer to maturity—in order to smooth consumption. However, such a back-loaded schedule is expensive, since later payments carry a higher default risk. To reduce borrowing costs, optimally the country will also shortens maturity. When calibrated to Brazilian data, the model can rationalize the observed issuance behavior, as an optimal trade-off between consumption smoothing and endogenous borrowing costs. (JEL E32, F34, G15, H63)

Keywords: Sovereign default; Debt management; Maturity; Payment schedule; Emerging market

* **Acknowledgments** The authors thank Cristina Arellano, Juan Carlos Hatchondo, Ignacio Presno, Juan Sanchez for their comments, as well as participants in the Midwest Macro Meetings 2014, the North American Econometric Society Meeting 2014, the Society of Economic Dynamics 2014, the 2014 ASU Reunion conferences, and the Department of Economics workshop at the University of Rochester. We are especially grateful to Cristina Arellano and Ananth Ramanarayanan for sharing their code for processing Bloomberg data extracts.

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1 Introduction

At least since the work of Rodrik and Velasco (1999) on the maturity of emerging market external debt, international economists have puzzled over such countries' substantial issuance of short-term debt during crises. Short-term debt is particularly vulnerable to roll-over risk, in a way that hurts consumption smoothing. We argue that this is less puzzling than one might conclude based on the maturity choice alone. Countries also adjust the stream of promised payments to be more back-loaded, i.e., relatively larger payments are scheduled closer to maturity, while the smaller payments are due sooner. This "tilting" or "twisting" of payments over a set lifetime for a bond allows the sovereign to mitigate the downsides of short-term borrowing.

In this paper we introduce a parsimonious measure, the average growth rate of scheduled payments, to capture the timing and relative size of coupons and principals of sovereign debt. A higher growth rate induces a more back-loaded schedule. We document that countries react to recessions by increasing payment growth and by shortening maturity. During downturns, countries prefer to delay relatively larger payments, to better smooth consumption. However, a schedule with such a high payment growth is expensive, given that later payments carry a higher default risk. To reduce borrowing costs, the country optimally shortens maturity. This choice reflects the tension between debt burden in the short-run and endogenous default risk, due to lack of commitment, in the future.

To understand how emerging economies choose the maturity and, more importantly, the growth rate of scheduled payments for external debt, we explore the contract-level bond data of eleven emerging markets, from the *Bloomberg Professional* service, using panel/instrumental variable methods. We report two major findings. First, the payment growth rate is higher when output is lower or spreads are higher. This implies that promised payments are more back-loaded during downturns. Second, the maturity of newly issued bonds is shorter during such episodes, consistent with the evidence presented by Arellano and Ramanarayanan (2012) and Broner, Lorenzoni, and Schmukler (2013).

Our model extends the standard quantitative sovereign default framework by introducing a flexible choice of payment schedule. A small, open economy can issue state-uncontingent bonds in the international financial markets. Its government can choose to default over its debt, subject to a "punishment" induced by output loss and temporary exclusion from international markets. We depart from the literature by allowing the government to issue bonds with both different maturities and schedules. For example, the government may issue a 10-year, back-loaded (front-loaded) long-term bond. Before the bond matures, the government makes periodic payments that growth (shrink) over time, at an endogenously chosen rate.

The payment schedule and maturity of sovereign debt are determined by the balance of two incentives: smoothing consumption and reducing default risk. In order to smooth consumption, the sovereign would like to align payments with future output, i.e., larger payments ought to be scheduled in periods with higher expected output. Thus, a more back-loaded schedule is preferable during economic downturns, since the government can repay the bulk of its obligation in the future, when the economy is expected to eventually recover. Therefore, under the consumption-smoothing incentive, the growth rate of payments and current output should be negatively correlated.

The government must also take into consideration its default risk when picking the terms of its bonds, since high default risk in the future imposes high borrowing cost at issuance. A more back-loaded bond is particularly expensive during downturns. The reason is that such a contract specifies that most payments are to be made in the distant future, which subjects lenders to large losses if the government defaults in the meantime. To reduce borrowing cost while enjoying the consumption-smoothing benefit of back-loaded contracts, the government chooses a shorter maturity in economic downturns. Contracts with shorter maturity allow lenders to realize their investment returns sooner. Lenders therefore bear less default risk and offer a higher bond price.

We calibrate the model to match key moments for the Brazilian economy. Our model generates volatilities of consumption and trade balance similar to the data, while replicating key features of sovereign debt. The median maturity is about nine years in the model and ten years in the data. The median growth rate of payment is five percent in the data and six percent in the model, which implies that, on average, countries issue back-loaded bonds.

Most importantly, our model matches the cyclical behavior of issuance well. When the spread increases above its unconditional mean, maturity shortens from seven to three years, while the payment growth rate increases from three to eight percent. Alternatively, by conditioning on the GDP level, we find that the cyclical properties of issuance in both model and data are in agreement.

This paper makes two contributions. Empirically, we construct a parsimonious measure of payment schedule and document the role of back-loading for consumption smoothing during downturns. Most studies in the literature, such as those of Broner, Lorenzoni, and Schmukler (2013) and Arellano and Ramanarayanan (2012), address this margin by focusing on the portfolio composition of both short and long debt.

Theoretically, we model the endogenous choice of payment schedule and maturity. The literature models government debt either using a one-period bond or an exogenous payment schedule. A newer line of work studying long-term sovereign debt as in Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009) studies Macaulay duration using perpetuity bonds with front-loaded payment schedules. We relax the specification of bond contracts by allowing both front- and back-loaded payment schedules, together with a finite maturity.

This paper follows the large sovereign default literature started by the seminal work of Eaton and Gersovitz (1981) and the quantitative analyses of Aguiar and Gopinath (2006) and Arellano (2008). The study of debt relief via restructuring, following default, is undertaken by Benjamin and Wright (2009), Yue (2010), and D'Erasmo (2011). A branch of the literature has turned to the role of maturity choice and the evidence it provides, for example Aguiar and Amador (2013), Dovis and Bocola (2015), and Mihalache (2015). Aguiar, Chatterjee, Cole, and Stangebye (2016) provide a thorough review of the state of the literature. Our paper is closely related to Sanchez, Sapriza, and

Yurdagul (2014). They allow for an endogenous maturity choice and emphasize its consequences for debt dilution. In comparison, we emphasize the growth rate of payments and its role in accounting for puzzling cyclical issuance behavior.

2 Empirical Analysis

This section documents how the maturity and payment schedules of new issuances vary with underlying fundamentals, using bond-level data. Our key finding is that during financial distress the sovereign shortens maturity and schedules payments to be more back-loaded, i.e., they promise smaller payments in the near future and relatively larger ones later.

We study a sample of eleven emerging market sovereigns[1]: Argentina, Brazil, Mexico, Russia, Colombia, Uruguay, Venezuela, Hungary, Poland, Turkey, and South Africa. Using the *Bloomberg Professional* database, we extract information on external debt and construct promised cash flows. In the data, contracts are fairly diverse, employing several coupon types (e.g. fixed rate, zero coupon, step, variable, float), with or without the presence of a callable or sinkable option, with varying coupon payment frequencies (e.g. annual and semi-annual), and occasionally with collateral or third party guarantees. See appendix A.3 for detailed characteristics and the cyclical behavior of coupon payments. Since we focus on issuance, we must address the fact that countries usually issue several bonds in any one time period, each with its own rich characteristics, an inherently high-dimensional object. In order to facilitate aggregation of cash flows, both across bonds and over time, we introduce a new statistics, the growth rate of payments, which captures in a parsimonious way the dynamics of payments over the lifetime of the bond. Moreover, this measure has a natural counterpart in our quantitative exercise and, more broadly, in the literature on maturity choice for sovereign debt.

We focus on foreign-currency denominated bonds and exclude bonds with special features (e.g., collateralized), zero-coupon[2], or guarantees from international financial institutions such as the IMF. Since countries issue debt denominated in several currencies, we convert these flows to real United States Dollars using exchange rates provided by the IMF and the CPI series from the Bureau of Labor Statistics. LIBOR rates from EconStats.com are used whenever a bond specifies its coupon rate relative to such a reference rate. The currency choice for new issuances is a source of variability in ex-post real cash flows, via movements in real exchange rates. This implies that even in the case of a fix coupon rate, the most popular type of contract in the data, the economically relevant value of payments will evolve over time.

We document key facts about these bond-level issuance data, in relation to GDP and the spread series provided by Broner, Lorenzoni, and Schmukler (2013). The spreads are at a weekly frequency

[1] This is the same set of countries considered in Broner, Lorenzoni, and Schmukler (2013)

[2] Most countries, except for Argentina, Turkey, and Uruguay, did not issue such bonds. In our sample, 40% of Argentina's bonds are zero-coupon, 84% for Uruguay and 12% for Turkey. In Appendix B, we document the robustness of our results to the inclusion of zero-coupon bonds.

and measured by the differences in the (annualized) yield-to-maturity relative to equivalent U.S. (or German) bonds. Their yield curve estimates deliver spread for bonds of maturities up to three years, between six and nine years, and over nine years. Appendix A contains further information on the data used.

2.1 Payment Schedule, Maturity, and Duration

We start by defining key concepts. We characterize bonds using three measures: maturity (T), Macaulay duration (D), and the growth rate of payments (δ), where the latter reflects both the coupon and principal payments. Consider a sovereign country i in period t. Let $c_t^i(s)$ denote the cash flow—in real US dollar terms—promised by the portfolio issued at period t to be paid $s \in \{1, 2, \ldots, N_t^i\}$ periods later. N_t^i refers to the number of periods until the last payment is scheduled. Let n be the number of periods in a year. For example, if the payment frequency is one quarter, we set $n = 4$ and s counts over the quarters in the lifespan of the bonds. This will be the time aggregation employed throughout our analysis. We consider an alternative case, yearly aggregation with $n = 1$, and check for robustness in Appendix B.

Whenever multiple bonds are issued during a given time period, e.g. in the same week, we sum over the cross-section of promised cash flows, at each future period, resulting in a single stream of payments $c_t^i(s)$, as if the country had issued a single bond making all the payments scheduled by the actual bonds issued. Such constructed streams are assigned a maturity T_t^i (measured in terms of years since the issue date) given by the average maturity of the actually issued bonds, weighted by each bond's real principal value. We label the promised cash-flow profile $\{c_t^i(s)\}_{s=1}^{N_t}$ as "payment schedule." To compute the annualized growth rate of payment δ_t^i, we regress the promised cash flows over the number of years elapsed since the issuance date t,

$$\log c_t^i(s) = \text{constant} + \delta_t^i \, \frac{s}{n} + \epsilon_t^i(s) \tag{1}$$

where $\epsilon_t^i(s)$ is an error term reflecting deviations of the actual schedule from a constant growth rate stream. Table 1 reports country-level, average R-squared statistics for these regressions.

Duration D_t^i measures the average length of time to payment, weighted by each payment. It is given by

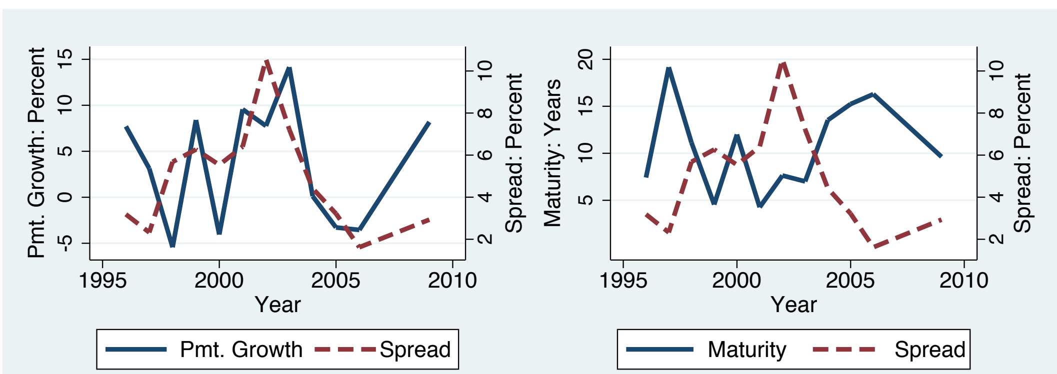
$$D_t^i = \sum_{s=1}^{N_t^i} \frac{c_t^i(s)\, R^{-s/n}}{\sum_{s=1}^{N_t^i}\{c_t^i(s)\, R^{-s/n}\}} \, \frac{s}{n}, \tag{2}$$

where R denotes the gross annual, real, risk-free rate, which we fix at 3.2 percent, following Arellano and Ramanarayanan (2012). Thus, D_t^i represents the risk-free version of the Macaulay duration and is referred to simply as the "duration." This is the measure commonly studied in the literature. Although this measure reflects both maturity and the payment schedule, it obscures their independent roles for issuance choice, as documented in our analysis.

Table 1 reports the summary statistics for the eleven countries in our sample. Most issued a large number of bonds, with a total of 935 bonds in our sample, six bond per country per year on average. The mean payment growth rate is 19 percent under a weekly issuance, while the average maturity is about nine years, and the average duration is about seven years. All countries issue back-loaded bonds, with positive payment growth, ranging from 11 percent to around 37 percent. The countries in our sample face high interest rate spreads, 3.4 percent on average.

We are interested in how emerging markets vary issuance characteristics with the business cycle, as reflected in the 6-to-9 year interest rate spread, labeled "9-year spread." We use the series from the yield curve estimation of Broner, Lorenzoni, and Schmukler (2013). Table 2 provides correlation coefficients of bond characteristics with the 9-year spread. For all countries except Russia, the maturity is negatively correlated with the spread, with correlations ranging from -0.06 to -0.48. Payments schedules are back-loaded when the interest rate is high, as reflected in their positive correlation for most countries. In addition, the correlation coefficients are similar for quarterly aggregation or yearly aggregation of payments, both with an average around 0.2.

Figure 1: Maturity, Payment Growth, and Spread (Brazil)



Note: Pmt. growth denotes the growth rate of payments for annual new issuances.

We illustrate the dynamics of payment growth and maturity in relation to the spread for the case of Brazil. The left panel of Figure 1 plots the spread and the payment growth with a yearly issuance, and the right panel plots the spread and the maturity. Brazil experienced an external and domestic debt crisis in 2002, when the interest spread increases to 15 percent, the new issuances are more back-loaded but of lower maturity. Overall the growth rate of scheduled payments co-moves with the spread, with a correlation of 0.40.[3] On the other hand, the maturity has a negative correlation of -0.48.

[3]The correlation is lower than the one in Table 2 due to a different issuance frequency.

Table 1: Summary Statistics

	Number of Bonds	Maturity (T, Years)	Pmt. Growth (δ, Percent)	Duration (D, Years)	Spread (r, Percent)	$R-$sq
Argentina	187	6.38	38.21	5.06	5.95	0.39
Brazil	78	11.43	12.57	7.04	4.31	0.24
Colombia	107	7.56	33.96	5.46	4.1	0.4
Hungary	29	8.71	15.57	7.32	0.94	0.25
Mexico	107	9.82	14.15	6.77	2.35	0.24
Poland	63	12.53	13.13	8.89	0.66	0.21
Russia	22	9.93	16.17	6.83	6.31	0.22
South Africa	25	9.87	14.09	7.25	1.99	0.22
Turkey	177	7.04	30.74	5.25	4.31	0.34
Uruguay	85	9.77	10.39	7.1	2.16	0.16
Venezuela	50	10.82	10.69	6.98	4.06	0.26
Average	85	9.44	19.06	6.72	3.38	0.27

Note: this table provides characteristics of weekly issues of bonds, excluding zero-coupon bonds. The length of a scheduled payment period is assumed as one quarter. $R-$sq refers to the $R-$squared from the estimation of the payment growth (δ). Pmt. Growth (δ) refers to the annual growth—in percentage points—in the payment schedule, and Spread (r) is the annual percentage (nine-year maturity) interest rate spread. Sample periods are: July 1993 - Dec 2001 for Argentina, July 1994 - June 2009 for Brazil, January 1993 - June 2009 for Colombia, February 1990 - June 2009 for Hungary, January 1991 - June 2009 for Mexico, October 1994 - June 2009 for Poland, January 1993 - June 2009 for Russia, October 1991 - June 2009 for South Africa, January 1990 - June 2009 for Turkey, January 1993 - Dec 2001 for Uruguay, and July 1991 - June 2009 for Venezuela.

Table 2: Cyclical Behavior of Bond Characteristics

Corr. with Spread	Maturity (T)	Duration (D)	Pmt. Growth (δ) Quarterly	Pmt. Growth (δ) Yearly
Argentina	-0.31	-0.30	0.10	0.08
Brazil	-0.48	-0.55	0.52	0.45
Colombia	-0.45	-0.52	0.42	0.39
Hungary	-0.35	-0.37	0.17	0.19
Mexico	-0.23	-0.30	0.23	0.23
Poland	-0.28	-0.34	0.30	0.32
Russia	0.32	0.25	-0.14	-0.01
South Africa	-0.06	-0.12	0.22	0.22
Turkey	-0.18	-0.21	0.00	-0.07
Uruguay	-0.50	-0.54	0.44	0.65
Venezuela	-0.12	-0.18	0.20	0.16
Average	-0.24	-0.29	0.22	0.24

Note: this table provides correlation coefficients of variables with the spread. Spread (r) is the annual percentage (nine-year maturity) interest rate spread. “Quarterly” refers to quarterly aggregation of payment and and “Yearly” refers to yearly aggregation of payment.

2.2 Regression Analysis

Given the suggestive correlations reported above, we undertake a more systematic analysis of the data by employing the specification introduced by Broner, Lorenzoni, and Schmukler (2013). We fix a weekly issuance frequency and regress bond characteristics on the interest rate spread, controlling for the real exchange rate, terms of trade, and the investment grade dummy, indicating whether the sovereign bond is rated as investment grade by credit rating agencies. More specifically, all dependent and explanatory variables are six-month moving averages (using a 26-week rolling window). All explanatory variables are in logs; the interest rate spread is in log-spread $\log(1 + r)$, multiplied by one hundred, where r refers to the actual level of the spread for the bond with the three-, nine- and twelve-year maturity, respectively.[4]

The OLS estimates are likely to be biased due either to unobserved country credit quality affecting both the spread and issuance characteristics (i.e., an omitted variables problem) or to the fact that the choice of bond characteristics might impact the spread itself (i.e., a reverse causality problem). Moreover, spreads are estimated rather than directly observed so that spreads are often imputed for some weeks, which adds unobserved measurement error. To address these issues, we use the instrumental variable (IV) method. The challenge is to find a valid instrument that is correlated with the spread and uncorrelated with unobserved drivers of bond characteristics. Since each of the countries in our sample has a negligible influence on financial conditions in financial centers, we use investors' risk appetite as an instrument for country spread, that is, the variation in borrowing terms induced by conditions in the center, exogenous to the country's circumstances.

Our main instrument is the US high-yield corporate bond index introduced by Broner et al., which reflects the spread for corporate bonds of a relatively low credit quality, below investment-grade. We argue that it is a valid proxy for the risk appetite of investors in the global bond markets, since it is plausibly unaffected by the bond supply behavior of countries in our sample. One potential concern is that the high-yield index could be affected by a latent factor common to all risky issuers, be they US corporations or emerging markets, rather than by the investors' attitude towards risk. As a robustness check, we use the spread of Moody's Aaa-rated corporate bonds index, which exhibit very low levels of, and volatility in, historical default rates. Therefore, the variation in the spread of these safe corporate bonds over time is, as is well documented in the literature, mainly accounted for by the investors' risk appetite rather than by default risk.

Table 3 reports our estimates. In all specifications, financial conditions are statistically significant determinants of issuance choice, with a positive coefficient in maturity regressions and negative in payment growth regressions. Throughout, results for duration are similar to those for maturity. The OLS and IV coefficients share the same sign but the IV coefficients are more precisely estimated. In addition, for a given dependent variable, estimates of the two different instrumental variables are

[4]The three-year spread is for bonds of maturities up to three years, nine-year spread reflects maturities between six and nine years, and twelve-year spread represents all bonds with maturities over nine years.

close to each other.

The magnitudes of these results are also economically significant. For every one percentage point increase in the spread, emerging markets will raise the growth rate of payments by four to six percentage points, i.e., back-load the schedule to a greater extent, and reduce maturity by about one year. These results are in line with the raw correlations presented previously. Since spreads are quite volatile, for example, varying between two and ten percent for Brazil, our findings imply substantial variation in these debt characteristics over the cycle.

To construct cash flows and estimate their payment growth δ we require an explicit choice of issuance frequency. To minimize any bias induced by time aggregation and to ensure consistency with the Broner, Lorenzoni, and Schmukler (2013) methodology, we use a weekly issuance frequency for our empirical section. This means we aggregate all bonds issued within one week into a composite bond that schedules all the payments of the individual bonds. In Appendix B, we document that our empirical results are robust to the choice of frequency for issuance, as well as to the frequency of payments and the inclusion of zero-coupon bonds.

Our analysis highlights two main results. First, the maturity of bonds shortens during periods of financial distress. This is consistent with the existing work on maturity choice of emerging markets. To the best of own knowledge, our second finding is new to the literature: sovereigns also adjust payment schedules in response to financial distress, by issuing more back-loaded bonds.

3 Model

We study optimal maturity and payment schedule of sovereign debt in a small, open economy model with default. A benevolent government borrows from a continuum of competitive lenders by issuing uncontingent debt with a flexible choice of maturity and payment schedule. The debt contract has limited enforcement, in that payments are state-uncontingent and the sovereign government has the option to default.

3.1 Technology, preferences, and international contracts

The economy receives a stochastic endowment y, which follows a first-order Markov process. The government is benevolent, and its objective is to maximize the utility of the representative consumer given by,

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where c_t denotes consumption in period t, $0 < \beta < 1$ the discount factor, and $u(\cdot)$ the period utility function, satisfying the usual Inada conditions. Each period, the government may borrow abroad by issuing a bond contract and decides whether to default on the outstanding debt. All the proceeds from bond issuance, net of payments on outstanding debts, are transferred as a lump sum to the

Table 3: Regression: Maturity, Duration, and Payment Growth

	Dependent Variable: Maturity (T)								
	OLS	IV: HY	IV: Aaa	OLS	IV: HY	IV: Aaa	OLS	IV: HY	IV: Aaa
3-y Spread	-0.32***	-0.71***	-0.41**						
	[0.11]	[0.15]	[0.17]						
9-y Spread				-0.76***	-0.73***	-0.42**			
				[0.14]	[0.15]	[0.17]			
12-y Spread							-0.57***	-0.72***	-0.42**
							[0.15]	[0.15]	[0.17]
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.03	0.04	0.02	0.08	0.05	0.02	0.07	0.05	0.02
	Duration (D)								
3-y Spread	-0.19***	-0.46***	-0.29***						
	[0.05]	[0.07]	[0.08]						
9-y Spread				-0.43***	-0.46***	-0.30***			
				[0.06]	[0.07]	[0.08]			
12-y Spread							-0.32***	-0.46***	-0.30***
							[0.07]	[0.07]	[0.08]
R^2	0.01	0.09	0.05	0.13	0.09	0.05	0.11	0.09	0.05
	Dependent Variable: Payment Growth Rate (δ)								
3-y Spread	2.08***	6.15***	3.33***						
	[0.75]	[0.89]	[1.03]						
9-y Spread				5.83***	6.24***	3.44***			
				[1.06]	[0.90]	[1.07]			
12-y Spread							4.46***	6.16***	3.44***
							[1.14]	[0.89]	[1.07]
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.04	0.08	0.03	0.12	0.08	0.03	0.10	0.08	0.03

Note: This table reports OLS and 2SLS (IV) regressions of weekly series of maturity, duration, and payment growth on spreads, controlling for real exchange rate, terms of trade, and investment grade dummy; the number of observations is 4405. All variables are 6-month moving averages and demeaned for each country. Control variables are in logs. Spreads are in log-spreads $log(1 + r)$, and multiplied by one hundred. 3-y Spread is for the bond with the maturity up to three years, 9-y Spread the maturity between six- and nine-years, and 12-y Spread the maturity greater than nine years. Either the US high-yield corporate bond index (HY) or the Moody's Aaa rated corporate bond spread relative to the 10-year Fed rate (Aaa) is used as an instrumental variable for spreads. Standard errors are robust to heteroskedasticity and within country serial correlation. ** significant at 5%; *** at 1%.

Table 4: First-Stage Regression of Spread

Dependent Variable	3-y Spread	3-y Spread	9-y Spread	9-y Spread	12-y Spread	12-y Spread
HY	2.91***		2.86***		2.90***	
	[0.17]		[0.13]		[0.13]	
Aaa		2.41***		2.33***		2.33***
		[0.15]		[0.11]		[0.12]
Controls	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.20	0.20	0.31	0.28	0.24	0.21

Note: This table reports the first-stage regression of weekly series of spreads on an instrumental variable, controlling for real exchange rate, terms of trade, and investment grade dummy. The number of observations is 6014. All variables are 6-month moving averages and demeaned for each country. Control variables are in logs. Spreads are in log-spreads $log(1+r)$, and multiplied by one hundred. 3-y Spread is for the bond with the maturity up to three years, 9-y Spread the maturity between six- and nine-years, and 12-y Spread the maturity greater than nine years. Either the US high-yield corporate bond index (HY) or the Moody's Aaa rated corporate bond spread relative to the 10-year Fed rate (Aaa) is used as an instrumental variable for spreads. Standard errors are robust to heteroskedasticity and within country serial correlation. ** significant at 5%; *** at 1%.

representative consumer. We assume the government has access to enough policy instruments[5] to be capable of perfectly control the overall national level of borrowing, thus avoiding any issues related to private sector over-borrowing, as discussed in Jeske (2006).

While in *good credit standing*, the government has the option to default on its debt. Following the sovereign default literature, we assume that after default, the debt is written off and the government switches to *bad credit standing*. The government is then subject to output losses and temporary exclusion from international financial markets. With probability ϕ, international lenders forgive a government in bad credit standing and resume lending to it.[6] Given default risk, lenders quote bond prices that compensate them for expected losses.

A bond contract specifies a maturity T and a payment schedule given by the growth rate of payments δ, the number of units issued b, and a bond price q. For such a contract, conditional on not defaulting, the government repays $(1+\delta)^{-\tau}$ with $0 \leq \tau \leq T$ periods to maturity. When δ is negative, the payments shrink over time (front-loaded).[7] When δ equals zero, the contract is "flat" as the payments are constant over T periods. When δ is positive, the payments grow over time (back-loaded). The contract also nests the zero coupon bond, when we let δ go to infinity. Figure 2 shows examples of schedules for different cases of δ, for ten-year bonds. To make contracts comparable, we pick the number of bond units issued b to finance one unit of consumption for all cases, using the risk-free bond price. With a more back-loaded schedule, the number of units issued b has to be larger due to discounting.

To mitigate the curse of dimensionality implicit in using richer descriptions of debt contracts, we assume that the government can only hold one type of bond at a time. If the government wants to change its payment schedule, it has to buy back the outstanding debt before it can issue a new contract. Under this assumption, the state of a government with good credit standing is $z = (T, \delta, b, y)$, including its income shock y and the outstanding units b, with remaining maturity T and growth rate of payments δ.

3.2 Equilibrium

The government's problem The government in good credit standing chooses whether to default d, with $d = 1$ denoting default:

$$V(z) = \max_{d \in \{0,1\}} \left\{ d\, V^d(y) + (1-d)\, V^n(z) \right\} \tag{3}$$

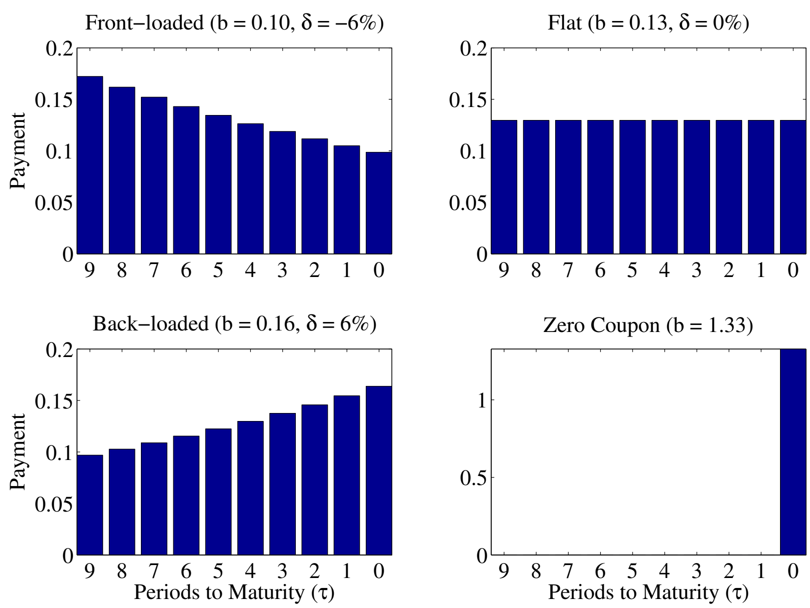
where V^d and V^n are the defaulting and repaying values respectively.

[5]For example, capital control policies, as studied by Kehoe and Perri (2004), Wright (2006), Kim and Zhang (2012), or Na, Schmitt-Grohé, Uribe, and Yue (2014).

[6]Our model abstracts from renegotiation. Yue (2010), D'Erasmo (2011), and Benjamin and Wright (2009) study debt renegotiation explicitly. Quantitatively, the predictions of such models in terms of standard business-cycle statistics of emerging economics are similar as that in Arellano (2008), without renegotiation.

[7]This is the case covered by the perpetuity bond in Arellano and Ramanarayanan (2012).

Figure 2: Payment schedule



Note: Payment schedules for bond contracts with different δ, against periods to maturity τ. The number of units issued b is picked so that all bonds finance 1 unit of consumption at the respective risk-free bond price.

If it defaults, the government gets its debt written off but receives a lower endowment $h(y) \leq y$. With probability ϕ, a government in bad credit standing will return to market, without any debt. The defaulting value satisfies

$$V^d(y) = u[h(y)] + \beta \mathbf{E} \left\{ (1-\phi) V^d(y') + \phi V(0,0,0,y') \right\}. \tag{4}$$

If it repays, the government can continue the current contract, with value V^c, or issue new debt and receive value V^r. We use $x = 0$ to denote continuing the current contract and $x = 1$ to denote issuing new debt. Specifically, the problem under no default is given by

$$V^n(z) = \max_{x \in \{0,1\}} \{x V^r(z) + (1-x) V^c(z)\} \tag{5}$$

where the value when continuing to service outstanding debt is

$$V^c(z) = u\left[y - \frac{b}{(1+\delta)^T}\right] + \beta \mathbf{E} V(T-1, \delta, b, y'), \tag{6}$$

and the value when choosing a new bond is

$$
\begin{aligned}
V^r(z) &= \max_{T',\delta',b'} \left\{ u(c) + \beta\, \mathbf{E}\, V(T',\delta',b',y') \right\} \\
\text{s.t. } c &= y - \frac{b}{(1+\delta)^T} + q(T',\delta',b',y)\, b' - q^{\mathrm{rf}}(T-1,\delta)\, b.
\end{aligned}
\tag{7}
$$

If it chooses to issue, the government must retire outstanding obligations, at the risk-free bond price q^{rf}. The proceeds from the sale of the new bond are $q(T',\delta',b',y)\,b'$, where the bond price schedule for new issuance, q, reflects future default risk and thus depends on the current endowment level y and the payment structure.

We assume that when buying back old bonds, the government faces a cost given by the risk-free bond price q^{rf}, the upper limit for the secondary-market price. This high cost is consistent with evidence on expensive buybacks discussed in Bulow and Rogoff (1988) and proxies for issuance costs in a reduced form. Here we abstract from issues of debt dilution, as studied by the recent literature on long-term sovereign debt, e.g., Hatchondo, Martinez, and Sosa-Padilla (2016) and Sanchez, Sapriza, and Yurdagul (2014). We conduct sensitivity analysis with respect to alternative buyback costs, allowing for dilution, in section 4.4.

International financial intermediaries Lenders[8] are risk neutral, competitive, and face a constant world interest rate r. The bond price schedule must guarantee that lenders break even in expectation. For a bond with remaining maturity T' and growth rate δ', its risk-free price is defined recursively as

$$
q^{\mathrm{rf}}(T',\delta') = \begin{cases} \dfrac{1}{1+r}\left[\dfrac{1}{(1+\delta)^{T'}} + q^{\mathrm{rf}}(T'-1,\delta)\right] & \text{for } T' \geq 1 \\ \dfrac{1}{1+r} & \text{for } T' = 0. \end{cases}
\tag{8}
$$

With default risk, lenders charge a higher interest rate to compensate for losses in the default event. For $T' \geq 1$, the bond price is therefore given by

$$
\begin{aligned}
q(T',\delta',b',y) = {} & \frac{1}{1+r}\, \mathbf{E} \left\{ (1 - d(T',\delta',b',y')) \times \right. \\
& \left. \left[\frac{1}{(1+\delta)^{T'}} + (1 - x(T',\delta',b',y'))\, q(T'-1,\delta',b',y') + x(T',\delta',b',y')\, q^{\mathrm{rf}}(T'-1,\delta') \right] \right\},
\end{aligned}
\tag{9}
$$

and for $T' = 0$ the bond price reduces to the usual one-period bond case

$$
q(0,\delta',b',y) = \frac{1}{1+r}\, \mathbf{E}\left\{ 1 - d(0,\delta',b',y') \right\}. \tag{10}
$$

[8] We assume that lenders have deep pockets and thus can unilaterally satisfy the country's loan demand. This rules out self-fulfilling crises due to lenders' failure to coordinate, as in Cole and Kehoe (2000).

The risky bond price reflects expected payments to lenders. If the government repays next period, lenders receive a payment of $(1+\delta)^{-T'}$ per unit outstanding. The repaying government may choose to restructure its debt $x' = 1$ and so repurchase its outstanding debt at the risk-free bond price q^{rf}. Note that maturity T' and payment schedule δ' affect the risky bond price in two ways: on one hand, conditional on no default, they matter for expected discounted payment and thus the risk-free component of q, the corresponding q^{rf}. On the other hand, both maturity and payment schedule matter for future default decisions and thus the default premium priced into q.

Definition of equilibrium The equilibrium consists of policy functions T', δ', b', d', x', value functions V, V^d, V^n, V^c, the bond price schedule q, and the risk-free schedule q^{rf}, such that, given the world interest rate r,

(a) policies and values satisfy the government's problem (3-7), given the bond prices, and

(b) lenders charge break-even bond prices (9) consistent with government policies, and the risk-free bond price schedule is given by (8).

4 Quantitative Analysis

We calibrate the model for the Brazilian economy over the period from 1996 to 2009 and study its implications for standard business cycle statistics and, most important, for the maturity and payment schedule of sovereign debt. We discuss the incentives faced by a country when designing its bond issuance. Finally, we conduct sensitivity analysis related to the cost of retiring outstanding debt and alternative shock specifications.

4.1 Calibration

We calibrate the parameter values of the model to match key moments in the yearly Brazilian data. The per-period utility function $u(c)$ exhibits a constant coefficient of relative risk aversion, σ,

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}. \tag{11}$$

The economy is subject to two, independent shocks: an endowment shock and a sudden stop shock. The endowment of this economy follows an AR(1) process

$$\log(y_t) = \rho \log(y_{t-1}) + \eta\, \varepsilon_t, \tag{12}$$

where the idiosyncratic shock ε_t follows the standard Normal distribution. Every period, with a constant probability p^{ss}, the country enters a *sudden stop state*, in which endowment is reduced and the country can only lower its debt burden. While in this state, the country has a constant probability p^{ret} of recovering in the next period.[9]

Following Arellano and Ramanarayanan (2012), the output of a country with a bad credit standing $h(y)$ is given by

$$h(y) = \min\{\{y,\ (1-\lambda_d)\,\mathbf{E}\,y\}\} \tag{13}$$

where $\mathbf{E}\,y$ is the unconditional mean of y and $\lambda_d \in [0,1]$ captures the default penalty. During sudden stop, the endowment is capped by $(1-\lambda_s)\,\mathbf{E}\,y$.

To compare model and data, we define the *yield to maturity* as the constant interest rate $\hat{r}$ such that the present value of payments computed using this interest rate is equal to the market price of the bond, i.e., $\hat{r}$ is implicitly defined by

$$q\left(T', \delta', b', y\right) = \sum_{\tau=T'}^{0} \exp\left[-\hat{r} \times \left(T' + 1 - \tau\right)\right] \frac{1}{\left(1+\delta'\right)^{\tau}}. \tag{14}$$

The *spread* is the difference between the yield to maturity $\hat{r}$ and the risk-free rate r:

$$\text{spread}\left(T', \delta', b', y\right) \equiv \hat{r}\left(T', \delta', b', y\right) - r. \tag{15}$$

Table 5: Benchmark Parameter Values

		Value	Target/Source
Parameters calibrated independently			
σ	Risk-aversion	2.0	Standard value
r	Risk-free rate	3.2%	Arellano and Ramanarayanan (2012)
ρ	Shock persistence	0.9	Arellano and Ramanarayanan (2012)
η	Shock volatility	0.017	Arellano and Ramanarayanan (2012)
ϕ	Prob. of return to market	0.17	Benjamin and Wright (2009)
p^{ss}	Prob. of sudden stop (s.s.)	0.10	Bianchi, Hatchondo, and Martinez (2012)
p^{ret}	Prob. of s.s. recovery	0.75	Bianchi, Hatchondo, and Martinez (2012)
Parameters calibrated jointly			
β	Discount factor	0.88	Jointly: Mean and standard deviation of 9y spread, median maturity, and the debt service to GDP ratio.
λ_d	Output loss due to default	5.0%	
λ_s	Output loss due to s.s.	-0.5%	
$\overline{T}$	Max. maturity	15	

Note: this table provides the benchmark parameter values used in calibrating the model.

Table 5 presents the calibrated parameter values. The risk-aversion parameter σ is set to two as is standard in the literature. The risk-free interest rate is set to 3.2 percent to target the average annual yield to maturity for US government bonds. The persistence and volatility of the AR(1) output

[9] For a version of the model with an explicit sudden stop state, see Appendix C.

process are taken from Arellano and Ramanarayanan (2012), who calibrate these two parameters to the HP-filtered Brazilian GDP. They pick $\rho = 0.9$ and compute the standard deviation $\eta = 0.017$. The probability of a defaulting country regaining access to the international financial market ϕ is set to 0.17, following Arellano and Ramanarayanan (2012). The annual probability of sudden stop p^{ss} and recovery p^{ret} are chosen to be 0.10 and 0.75, consistent with the quarterly values used by Bianchi, Hatchondo, and Martinez (2012). The four remaining parameters, the discount factor β, the output loss parameters λ_d and λ_s, together with the maximum maturity $\overline{T}$ are chosen jointly, to match the mean and standard deviation of the nine-year spread, median maturity, and the debt service to GDP ratio.

Table 6: Key Statistics: Data vs. Model

	Data	Baseline	No SS
Targeted Moments			
Mean 9-y Spread	4	4	4
Std 9-y Spread	3	4	4
Debt Service / GDP	5	5	6
Median Maturity	10	9	10
Other Moments			
Median Payment Growth	5	6	3
Mean 3-y Spread	5	5	2
Std 3-y Spread	4	6	4
Std C / Std Y	110	113	113
Std NX/Y / Std Y	36	55	56
Corr B/Y, Y	-53	-23	26
Corr Spread, Y	-49	-34	-33

Note: "No SS" refers to the calibrated model without sudden stop shocks. Std denotes standard deviation and Corr correlation. C is consumption, Y is GDP, NX is net export, B is total debt. Except for maturity, all values are expressed as percentages.

Table 6 compares the baseline model (column 2) and data (column 1) statistics for Brazil. The model matches the targeted moments well. For both data and model, we focus on new issuance.[10] The median maturity is ten years in the data and nine years in the model. The model also replicates payment growth and key business cycle features of emerging markets well. It predicts a six percent growth rate of payments, consistent with annual frequency data, where the median growth rate of payments is five percent[11], implying a back-loaded payment schedule for new issuance. In

[10] Following the sovereign default literature, for computational reasons, we restrict the sovereign to hold only one asset at a time. It then must be the case that this period's issuance will be next period's stock. In contrast, in the data the stock at any one time is the accumulation of many issuances, at various moments in the past. Faced with a choice between targeting stocks and matching flows (issuance), we follow the literature and study issuance, e.g., Arellano and Ramanarayanan (2012) and Broner, Lorenzoni, and Schmukler (2013).

[11] This number is different from the value reported in Section 2, where the analysis is conducted at a weekly

the model's limiting distribution, effectively no zero-coupon bonds are issued. It generates excess volatility of spreads relative to the data. Consumption is more volatile than output, as documented by Neumeyer and Perri (2005). The volatility of consumption is 1.1 times that of output in both the model and the data. The model produces a volatile trade balance (normalized by GDP), 55 percent in the model and 36 percent in the data. In Brazil, the spreads for all maturities are countercyclical. The correlations are -46, -53, and -48 percent for three-year, nine-year, and twelve-year spread with GDP, respectively. Table 6 reports the average of these correlations, -49 percent. This correlation is also negative in the model, -34 percent.

In the data the debt-to-GDP ratio is strongly countercyclical, with a correlation with GDP of -53 percent. To replicate this behavior, we need to include a sudden stop shock that creates an additional precautionary saving motive, discouraging excessive borrowing during good times when spreads are low. Eliminating the sudden stop increases the correlation of debt-to-GDP with GDP to 26 percent but leaves other moments relatively unchanged, as shown in the third column of Table 6.

4.2 Bond Price Schedule

The choice of optimal contracts depends on government's preferences and the bond price schedule it faces. This schedule depends on future governments' default incentives, which are determined by two channels: lack of commitment and debt burden. Contracts which make eventual default more tempting for the government (lack of commitment) or which require higher payments (debt burden), will carry higher default risk, lower prices and therefore be less attractive for debt finance.

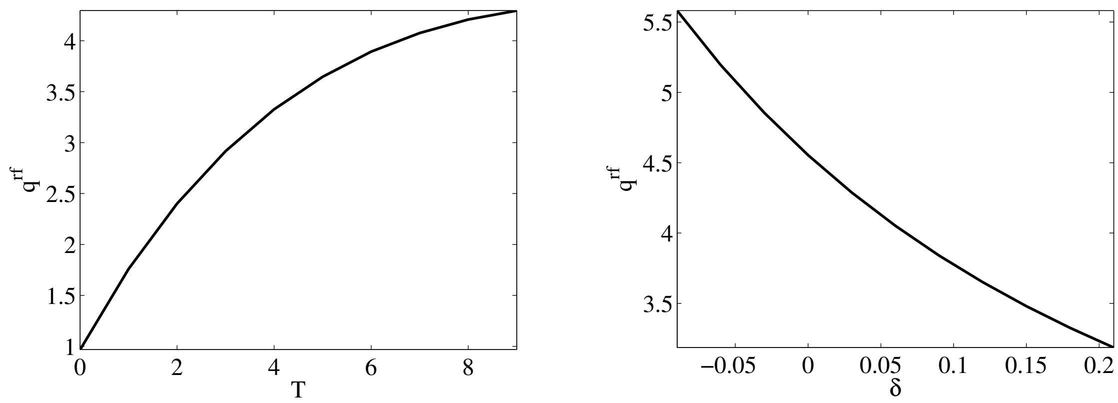
The bond price reflects the lender's opportunity cost, the equivalently structured risk-free bond price q^{rf}. This price varies with T' and δ', due to the changes they induce in the size and number of payments. All other contract characteristics constant, longer maturity implies more payments and thus a higher risk-free bond price. See Figure 3(a). A high δ' is associated with back-loaded payments, which are subject to compounded discounting and thus have lower present value, resulting in a lower risk-free price. See Figure 3(b).

To isolate the consequences of default risk, Figure 4 plots the market bond price schedule $q(T', \delta', b', y)$ relative to the risk-free bond price $q^{\text{rf}}(T', \delta')$ as a function of $q^{\text{rf}}(T', \delta')b$. We normalize the number of units b with q^{rf} to facilitate comparisons of debt values across different contracts. For any given T' and δ', issuing more units means a higher debt burden and thus higher risk of default and a lower bond price.

Figure 4(a) compares the bond price across growth rates of payments, $\delta = -3\%$ versus $\delta = 18\%$, for a fixed $T = 14$ and mean endowment. Suppose the government wants to issue debt with a given present, face value and that it is contemplating using the $\delta = 18\%$ contract, i.e., a more

frequency. For the purposes of our quantitative exercise we need to time-aggregate cash flows to yearly payments. We confirm that our empirical findings also hold for the yearly-aggregated payment schedule. (See Table 10 in Appendix B).

Figure 3: Risk-Free Bond Price q^{rf}



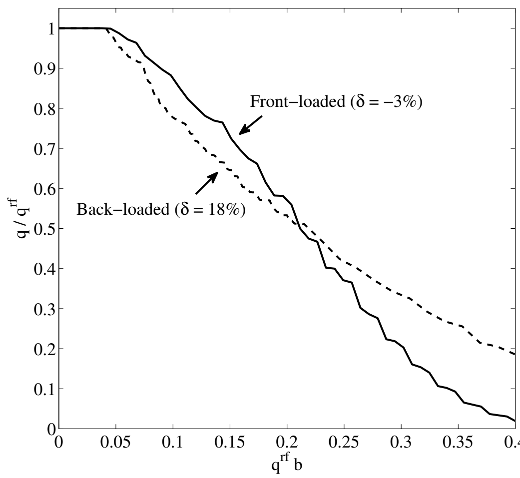
(a) Maturity (b) Payment Growth

back-loaded bond. This instrument schedules the larger payments later and smaller ones in the near future, leading to a lower debt burden and default risk in the short-term and a higher bond price. The downside of such an arrangement is that lenders will need to wait more for the bulk of payments, during which time fundamentals could worsen. As payments are pushed further into the future, conditional forecasts of the endowment process are less precise. This depresses bond prices, since lenders need to price these additional unfavorable contingencies. Which of these two effect dominates hinges on the level of debt issued. In Figure 4(a), for low levels of debt, such as 0.1, the front-loaded $\delta = -3\%$ offers better terms for borrowing. For higher debt levels, e.g. in excess of 0.3, back-loading becomes a better arrangement.

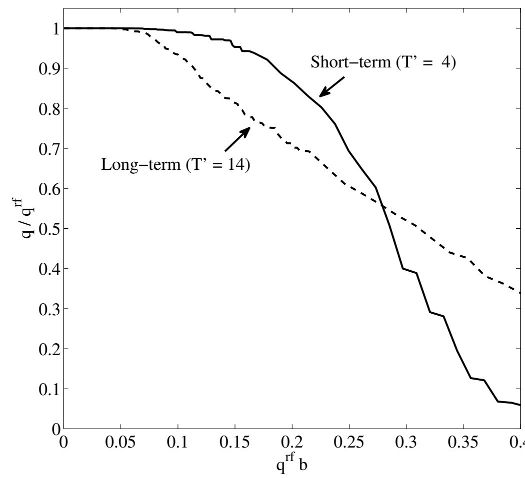
Figure 4(b) compares the bond price across maturity choices, $T = 4$ versus $T = 14$, for a fixed $\delta = 18\%$ and mean endowment. By lengthening maturity, with a constant present, face value of debt, the payments due in each period will be lowered since the debt is spread over a longer time horizon. This will tend to reduce default temptations. Such an arrangement, though, leaves lenders particularly vulnerable to future downturns. Not only are forecasts about the more distant future less precise, but if the country does default, the entire tail of the stream of payments is lost. This means that lenders would like the country to issue short-term debt and revisit markets frequently, so that current economic conditions are more informative. Under the case of Figure 4(b), the need to spread payments into the future will dominate for higher debt levels, e.g. over 0.3, while lower debt burden is best rolled-over with shorter maturities.

When maturity is short, the bond price schedule becomes insensitive to the choice of δ, as shown in Figure 4(c). This is because the two channels are fairly balanced. This suggests that when countries shorten maturity during recessions, they are likely to smooth consumption by back-loading payments because the adverse effect of such a pattern on the bond price is negligible. Figure 4(c)

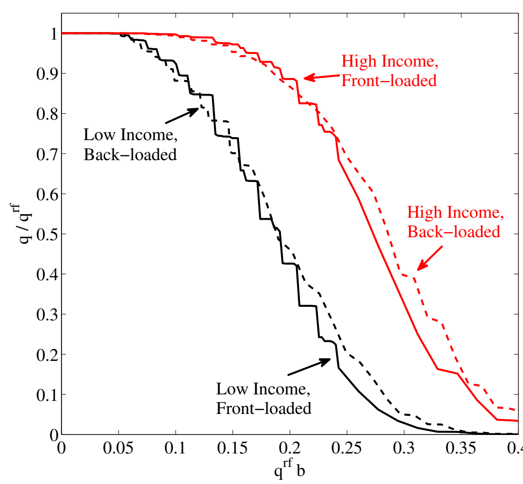
Figure 4: Bond Price Schedule



(a) $T = 14$, Mean Endowment (y)



(b) $\delta = 18\%$, Mean Endowment (y)



(c) $T = 4$

also illustrates the role of income in determining bond prices: higher income implies fewer incentives to default and thus the country can borrow more cheaply.

4.3 Maturity and Payment Schedule

We now turn our focus to understanding how maturity and payment structure of issuance vary with the business cycle. We use the spread and output as our preferred cyclical indicators. Table 7 reports key statistics for Brazil and their model counterparts. In the data, during normal times when the spread is below its historic mean, the growth rate of payments is about two percent, with a maturity of roughly 14 years and a duration of nine years. During periods of financial distress, when the spread is above average, payments become more back-loaded, with a growth rate of eight percent, maturity shortens to about seven years, and duration is reduced to six years. These patterns are consistent with the findings in Section 2, for a broader set of countries.

Our model matches the observed cyclical behavior of maturity, payment growth, and duration well. When the spread increases above its mean, the payment growth rate increases from three to eight percent, while maturity shortens from seven to about three years, and the duration decreases from five to three years. Even though the model generates a lower maturity and duration, on average, relative to the data, it successfully matches the overall pattern of the changes: in both data and model maturity decreases when spreads increase. Using GDP as a cyclical indicator, we get a similar message. When GDP is below trend, the country shortens maturity from 12 to nine years but back-loads the payment to eight percent. Duration follows the dynamics of maturity. In particular, it shortens by about three years.

The payment schedule and maturity of sovereign debt are determined by the interplay of two incentives: (i) smoothing consumption, and (ii) lowering the borrowing cost by reducing default risk. To smooth consumption, the sovereign would like to align payments with future output, i.e. scheduling larger payments for periods with higher expected output. Given the mean-reverting nature of the output process considered, the growth rate of output decreases with its current level. Thus, a more back-loaded schedule is preferable during economic downturns since the government can repay the bulk of its obligation in the future, when the economy is expected to recover. Under the consumption-smoothing incentive, the growth rate of payments and current output should be negatively correlated.

The government also takes into consideration the borrowing cost it faces when making choices over payment schedules. During downturns, when income is low, the range of debt levels for which back-loaded contracts offer better bond prices shrinks, as Figure 4(b) shows. This makes the sovereign more likely to face a tighter bond price if it were to choose a more back-loaded contract. To reduce the borrowing cost, while enjoying the consumption-smoothing benefit of more back-loaded contracts, the government chooses a shorter maturity in downturns to mitigate its lack of commitment. Moreover, for short maturities, the differences in bond price schedules for different payment growth rates are small, as Figure 4(c) shows. In summary, the short maturity reduces the overall level of the borrowing cost as well as the adverse effect of back-loading on the bond price, where the latter is especially attractive to a borrower during downturns.

4.4 Sensitivity Analysis

In this section, we conduct sensitivity analysis for cases without sudden stop shock and alternative bond buyback prices. We first recalibrate the model assuming no sudden stop shock. In particular, the median maturity is calibrated to be 10 years as in the baseline model. The third column of

Table 7: Payment Growth, Maturity, and Duration: Cyclical Properties

	Spread			GDP		
	Below Mean	Above Mean	Δ	Below Mean	Above Mean	Δ
Payment Growth (δ, %)						
Data	2	8	6	8	-0.5	-8.5
Baseline	3	8	5	9	2	-7
No SS	1	4	3	5	-2	-7
P. Dilution	7	9	2	9	6	-3
Maturity (T, Years)						
Data	14	7	-7	9	12	3
Baseline	7	3	-4	3	11	8
No SS	8	5	-3	6	11	5
P. Dilution	6	3	-3	2	12	10
Duration (D, Years)						
Data	9	6	-3	6	8	2
Baseline	5	3	-2	3	7	4
No SS	6	4	-2	5	7	2
P. Dilution	5	3	-2	3	9	6

Note: All "Data" values are based on annual issuance and refer to medians conditional either on spread or GDP. Payment growth (δ, %) is annualized percentage growth in the quarterly payment schedule. "No SS" and "P. Dilution" refer to the model without the sudden stop shock and the one with a partial dilution buyback price respectively.

Table 6 shows that the model without sudden stop shocks generates similar volatilities of spread and net export as in the baseline. The sudden stop shock, however, matters for characteristics of debt issuance. The sovereign issues fewer back-loaded bonds, the median payment growth is reduced to three percent, compared to six in the baseline model. This is mainly driven by borrowing choices of high-income states, under which the sovereign tends to issue relatively front-loaded bonds, if it borrows. Without sudden stop shock, high-income states have a more modest precautionary motive and thus borrow more. We therefore observe lower median payment growth and positively correlated debt-to-GDP and GDP.

Though having a low payment growth, the model without sudden stop still produces counter-cyclical payment growth and procyclical maturity issuance. When the interest rate spread becomes high (i.e., GDP is below trend), the payment growth rate of new issuance increases by two percent, and the maturity shortens by three years, as Table 7 shows. These findings are robust to the use of GDP as our conditioning variable.

We now turn to an alternative specification of bond buyback price. In our main analysis we used the risk-free bond price q^{rf} to retire outstanding debt, thus abstracting from any issues raised by long-term debt dilution. First, we consider the "full dilution" case with buyback at the competitive, secondary market price. This price results from valuing outstanding debt using the default probabilities implied by new issuance. The logic is that if the government retires all but a measure zero of outstanding bonds, these bonds' remaining payments would be subjected to the same default risk as the newly issued bond. This makes the buyback price a function of both current state variables (T, δ, y) and issuance characteristics (T', δ', b'). The full dilution bond price q^{fd} is given by

$$\begin{aligned} q^{\mathrm{fd}}\left(T, \delta, y, T', \delta', b'\right) = & \frac{1}{1+r}\, \mathbf{E}\left(1 - d'\left(T', \delta', b', y'\right)\right) \Big\{ (1+\delta)^{-T} \\ & + x\left(T', \delta', b', y'\right)\ q^{\mathrm{fd}}\left(T-1, \delta, y', T'', \delta'', b''\right) \\ & + \left(1 - x\left(T', \delta', b', y'\right)\right)\ q^{\mathrm{fd}}\left(T-1, \delta, y', T'-1, \delta', b'\right) \Big\} \end{aligned} \tag{16}$$

where $\langle T'', \delta'', b'' \rangle$ are the optimal choices in state $\langle T', \delta', b', y' \rangle$, conditional on restructuring. Consistent with Sanchez, Sapriza, and Yurdagul (2014), we find that, under full dilution, short-term debt strictly dominates and only one period bonds are issued in the ergodic distribution of the model. This is clearly inconsistent with the data. Sanchez, Sapriza, and Yurdagul (2014) show that with the introduction of sudden stop shocks, a higher level of risk aversion, or a debt restructuring procedure can revert this extreme result.

Given the lack of variation in optimal maturity under full dilution, we study a hybrid case, labeled "partial dilution," in which the buyback price is an average of the risk-free price and the full dilution price. The partial dilution price is given by

$$q^{\mathrm{pd}}\left(T, \delta, y, T', \delta', b'\right) = \frac{1}{2}\left[q^{\mathrm{rf}}\left(T, \delta\right) + q^{\mathrm{fd}}\ \left(T, \delta, y, T', \delta', b'\right)\right]. \tag{17}$$

For our numerical results, we keep maximum maturity $\overline{T} = 15$ as in the baseline, and recalibrate other parameters. The partial dilution model can deliver cyclicality results in line with our baseline and the data. However, on average, this produces shorter maturity and higher payment growth on average, relative to baseline. The overall effect of dilution is to shorten maturity and increase back-loading of payments. This level effect leaves intact our key findings, in terms of the magnitude of changes in issuance characteristics, with respect to spread or GDP. See Table 7.

5 Conclusion

In this paper we address the outstanding puzzle of short-term borrowing by emerging markets during crises. We go beyond the previous characterization of debt in terms of duration and instead consider two complementary measures: payment growth rate and maturity. In our model, as in the data, countries in crisis issue bonds with back-loaded payments and shorter maturity. This renders the choice of maturity less of a puzzle, given that such a schedule helps with risk-sharing during downturns. Our specification of long-term debt could prove a natural fit for studying the funding decision for long-term investment projects (e.g., infrastructure), where back-loaded debt might be more attractive, compared to the usual front-loaded schedules of the decaying perpetuity bond.

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A Data Appendix

A.1 Exchange Rate, U.S. CPI, and LIBOR

Sovereigns often schedule payments over the course of 20 or 30 years in the future since the issue date. In order to evaluate such promised payments in terms of real U.S. dollars, several assumptions are necessary:

- Exchange Rate: Under the assumption that foreign exchange rates are Martingales, the expected future exchange rate is equal to the current value.
- U.S. CPI: For the U.S. CPI, we assume perfect-foresight because the U.S. CPI is quite stable.
- LIBOR: When the coupon rate is expressed as a spread over the LIBOR rate, e.g., the floating coupon-rate bond, we take as our benchmark the perfect-foresight case in measuring the LIBOR rates in the future.

Note that our sample includes bonds with non-fixed coupon rate, e.g., floating and variable coupon-rate bonds, as well as the fixed coupon-rate bond. By contrast, frequently in the literature, non-fixed coupon-rate bonds are excluded from the analysis mainly for convenience rather than for economic reasons. We must address all of these cases consistently to produce a coherent picture of payment timing and size. For example, a variable coupon bond often specifies that coupon rates rise with the length of time to payments in a step-wise form; this has important implications for the growth rate of promised payments, i.e., positive growth rate of promised payments.

A.2 Sample Selection: Excluding Bonds with Special Features

We exclude from the sample bonds that are either denominated in local currencies or of special features for the reason explained in Broner, Lorenzoni, and Schmukler (2013). First, we focus on bonds denominated in foreign currencies for following reason. In many cases for emerging market economies, sovereign bonds are denominated in foreign currencies. Sovereigns do issue bonds denominated in their local currencies; in such a case, sovereigns would have an option to dilute their debt burden by adjusting the inflation rate in local currency terms, which is not the case for the bonds denominated in foreign currencies and is ruled out by the standard sovereign-default models, such as the one studied in this paper. [12] Thus, we simply focus on foreign-currency denominated bonds by excluding local-currency denominated bonds from our sample. Second, for the same reason as above, we exclude from the sample bonds with special features that are absent in our model and infrequently observed in the data: for instance, we exclude either collateralized

[12]Moreover, as discussed in Broner, Lorenzoni, and Schmukler (2013), if both foreign- and local-currency denominated bonds were included in the sample, then the regression analysis of bond characteristics would require controlling for the time-varying exchange-rate risk premium, which is difficult to measure.

Table 8: Statistics of Coupon Rate (%)

Country	Obs	Mean	Std	Min	Max	Corr w/ Spread
Argentina	132	7.99	3.29	0.00	13.49	-0.50
Brazil	66	9.34	2.59	3.76	20.12	0.07
Colombia	82	8.68	2.79	0.00	13.43	0.22
Hungary	23	4.14	1.54	0.82	6.56	0.45
Mexico	78	7.52	2.39	2.25	14.25	0.33
Poland	43	4.30	1.72	0.84	7.75	0.51
Russia	12	8.89	3.00	2.99	12.75	0.17
South Africa	21	6.83	2.49	2.00	10.50	0.12
Turkey	129	7.99	2.56	1.67	13.86	0.53
Uruguay	45	6.24	1.37	2.50	8.75	0.33
Venezuela	37	8.87	2.83	0.00	13.92	0.24
Average	61	7.34	2.41	1.53	12.31	0.22

Note: this table provides statistics of the coupon rate (in percentage points) of weekly issued bonds. Coupon rate is measured as the ratio of the per-year nominal value of coupon payments to the nominal principal value. Std refers to the standard deviation, and Corr with Spread the correlation coefficients of the coupon rate with the spread, where the spread refers to the six-to-nine year maturity interest rate spread.

bonds or bonds with the special guarantees provided by the third-party institutions such as the *IMF*, *World Bank*, and leading foreign governments/banks.

A.3 Coupon Rate

Table 8 provides statistics of the annualized coupon rate (measured as the ratio of the per-year nominal coupon payment to the nominal principal value), where the issuance period is still weekly; if multiple bonds are issued for a given week, then we average over coupon rates (weighted by the bond's real principal value) similarly to maturity. The average coupon rate is about seven percent, with a coefficient of variation of 33 percent. The coupon rate is positively correlated to the spread. In Table 9 we confirm the robustness of this association by using the coupon rate as the dependent variable in the two-stage least-squares specification we employ to study all other bond characteristics.

Table 9: Regression of Coupon Rate

	Dependent Variable: Coupon Rate								
	OLS	IV: HY	IV: Aaa	OLS	IV: HY	IV: Aaa	OLS	IV: HY	IV: Aaa
3-y Spread	0.15***	0.31***	0.29***						
	[0.04]	[0.06]	[0.06]						
9-y Spread				0.26***	0.31***	0.30***			
				[0.05]	[0.06]	[0.06]			
12-y Spread							0.19***	0.31***	0.30***
							[0.04]	[0.06]	[0.06]
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.15	0.16	0.14	0.19	0.16	0.14	0.17	0.16	0.14

Note: This table reports OLS and 2SLS (IV) regressions of the coupon rate on spreads, controlling for real exchange rate, terms of trade, and investment grade dummy; the number of observations is 4405. All variables are 6-month moving averages and demeaned for each country. Control variables are in logs. Spreads are in log-spreads $log(1+r)$, and multiplied by one hundred. 3-y Spread is for the bond with the maturity up to three years, 9-y Spread the maturity between six- and nine-years, and 12-y Spread the maturity greater than nine years. Either the US high-yield corporate bond index (HY) or the Moody's Aaa rated corporate bond spread relative to the 10-year Fed rate (Aaa) is used as an instrumental variable for spreads. Standard errors are robust to heteroskedasticity and within country serial correlation. ** significant at 5%; *** at 1%.

B Robustness of Empirical Findings

In this appendix we confirm the robustness of our results to alternative assumptions about the frequency of payments over the lifetime of the bond, the frequency with which bonds are issued, and with respect to our sample selection criteria.

Table 10 reports the estimates for our IV specification when bond payments are made yearly (i.e. we aggregate all individual, scheduled payments within each calendar year) as opposed to quarterly, our reference case. The frequency of issuance is kept at weekly. This alternative time aggregation assumption changes our estimates little, compared to the reference results in Table 3.

The main estimates are also robust to the use of a monthly issuance frequency, i.e. treating all bonds issued within the month as one, consolidated cash flow, as documented in Tables 11 and 12. Coefficients preserve their statistical significance and sign.

Finally, we expand our sample to include zero-coupon bonds, since a subset of countries issue a non-negligible number of such bonds. Tables 13 and 14 document that all coefficients of interest preserve their sign and significance. The OLS and IV results for the reference instrument (CSFB HYI) are unaffected. The alternative instrument (Moody's Aaa) loses significance and magnitude in the equation for payment growth.

Table 10: Regression of Payment Growth: Case of the Yearly Payment Schedule (Weekly Issuance)

	Dependent Variable: Payment Growth Rate (δ)								
	OLS	IV: HY	IV: Aaa	OLS	IV: HY	IV: Aaa	OLS	IV: HY	IV: Aaa
3-y Spread	2.40***	7.30***	4.78***						
	[0.77]	[1.00]	[1.21]						
9-y Spread				5.57***	7.42***	4.94***			
				[0.96]	[1.02]	[1.25]			
12-y Spread							4.38***	7.32***	4.93***
							[1.04]	[1.01]	[1.25]
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.03	0.08	0.03	0.09	0.08	0.03	0.08	0.08	0.03

Note: For weekly issuance of bonds, the yearly payment schedule and its estimated payment growth are considered. This table reports OLS and 2SLS (IV) regressions of payment growth on spreads, controlling for real exchange rate, terms of trade, and investment grade dummy; the number of observations is 4405. All variables are 6-month moving averages and demeaned for each country. Control variables are in logs. Spreads are in log-spreads $log(1+r)$, and multiplied by one hundred. 3-y Spread is for the bond with the maturity up to three years, 9-y Spread the maturity between six- and nine-years, and 12-y Spread the maturity greater than nine years. Either the US high-yield corporate bond index (HY) or the Moody's Aaa rated corporate bond spread relative to the 10-year Fed rate (Aaa) is used as an instrumental variable for spreads. Standard errors are robust to heteroskedasticity and within country serial correlation. ** significant at 5%; *** at 1%.

Table 11: Regression: Maturity, Duration, and Payment Growth: Case of Monthly Issuance

	Dependent Variable: Maturity (T)								
	OLS	IV: HY	IV: Aaa	OLS	IV: HY	IV: Aaa	OLS	IV: HY	IV: Aaa
3-y Spread	-0.34***	-0.73***	-0.41**						
	[0.13]	[0.18]	[0.20]						
9-y Spread				-0.79***	-0.74***	-0.43**			
				[0.15]	[0.18]	[0.21]			
12-y Spread							-0.62***	-0.73***	-0.43**
							[0.17]	[0.18]	[0.21]
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.03	0.04	0.02	0.09	0.04	0.02	0.08	0.04	0.02
	Duration (D)								
3-y Spread	-0.20***	-0.47***	-0.30***						
	[0.07]	[0.08]	[0.09]						
9-y Spread				-0.45***	-0.47***	-0.31***			
				[0.08]	[0.09]	[0.10]			
12-y Spread							-0.34***	-0.47***	-0.31***
							[0.09]	[0.08]	[0.10]
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.06	0.08	0.04	0.14	0.08	0.04	0.11	0.08	0.04
	Dependent Variable: Payment Growth Rate (δ)								
3-y Spread	1.91**	5.12***	2.16*						
	[0.89]	[1.10]	[1.29]						
9-y Spread				5.90***	5.18***	2.23*			
				[1.33]	[1.12]	[1.33]			
12-y Spread							4.59***	5.13***	2.24*
							[1.47]	[1.11]	[1.34]
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.02	0.06	0.01	0.11	0.06	0.01	0.09	0.06	0.01

Note: This table reports OLS and 2SLS (IV) regressions of *monthly* issuance characteristics on spreads, controlling for real exchange rate, terms of trade, and investment grade dummy; the number of observations is 1016. All variables are 6-month moving averages and demeaned for each country. Control variables are in logs. Spreads are in log-spreads $log(1+r)$, and multiplied by one hundred. 3-y Spread is for the bond with the maturity up to three years, 9-y Spread the maturity between six- and nine-years, and 12-y Spread the maturity greater than nine years. Either the US high-yield corporate bond index (HY) or the Moody's Aaa rated corporate bond spread relative to the 10-year Fed rate (Aaa) is used as an instrumental variable for spreads. Standard errors are robust to heteroskedasticity and within country serial correlation. ** significant at 5%; *** at 1%.

Table 12: First-Stage Regression of Spread: Case of Monthly Issuance

Dependent Variable	3-y Spread	3-y Spread	9-y Spread	9-y Spread	12-y Spread	12-y Spread
HY	2.93***		2.90***		2.92***	
	[0.23]		[0.18]		[0.18]	
Aaa		2.49***		2.41***		2.40***
		[0.20]		[0.15]		[0.16]
Controls	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.22	0.22	0.34	0.32	0.24	0.22

Note: This table reports the first-stage regression of *monthly* series of spreads on an instrumental variable, controlling for real exchange rate, terms of trade, and investment grade dummy. The number of observations is 1386. All variables are 6-month moving averages and demeaned for each country. Control variables are in logs. Spreads are in log-spreads $log(1 + r)$, and multiplied by one hundred. 3-y Spread is for the bond with the maturity up to three years, 9-y Spread the maturity between six- and nine-years, and 12-y Spread the maturity greater than nine years. Either the US high-yield corporate bond index (HY) or the Moody's Aaa rated corporate bond spread relative to the 10-year Fed rate (Aaa) is used as an instrumental variable for spreads. Standard errors are robust to heteroskedasticity and within country serial correlation. ** significant at 5%; *** at 1%.

Table 13: Regression: Maturity, Duration, and Payment Growth: Inclusion of Zero-Coupon Bonds

	Dependent Variable: Maturity (T)								
	OLS	IV: HY	IV: Aaa	OLS	IV: HY	IV: Aaa	OLS	IV: HY	IV: Aaa
3-y Spread	-0.35***	-0.77***	-0.51***						
	[0.12]	[0.15]	[0.17]						
9-y Spread				-0.77***	-0.78***	-0.52***			
				[0.14]	[0.15]	[0.17]			
12-y Spread							-0.57***	-0.77***	-0.52***
							[0.15]	[0.15]	[0.17]
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.04	0.05	0.03	0.09	0.05	0.03	0.07	0.05	0.03
	Duration (D)								
3-y Spread	-0.21***	-0.48***	-0.33***						
	[0.06]	[0.07]	[0.08]						
9-y Spread				-0.44***	-0.49***	-0.34***			
				[0.07]	[0.07]	[0.08]			
12-y Spread							-0.33***	-0.48***	-0.34***
							[0.07]	[0.07]	[0.08]
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.08	0.09	0.06	0.14	0.09	0.06	0.12	0.09	0.06
	Dependent Variable: Payment Growth Rate (δ)								
3-y Spread	1.31**	4.84***	1.60						
	[0.65]	[0.87]	[1.04]						
9-y Spread				4.93***	4.91***	1.65			
				[1.02]	[0.89]	[1.07]			
12-y Spread							3.83***	4.85***	1.65
							[1.06]	[0.87]	[1.07]
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.02	0.06	0.02	0.08	0.06	0.02	0.07	0.06	0.02

Note: This table reports OLS and 2SLS (IV) regressions of weekly series of maturity, duration, and payment growth on spreads, controlling for real exchange rate, terms of trade, and investment grade dummy; differently from the main regression, zero-coupon bonds are now included into the sample. The number of observations is 4417. All variables are 6-month moving averages and demeaned for each country. Control variables are in logs. Spreads are in log-spreads $log(1+r)$, and multiplied by one hundred. 3-y Spread is for the bond with the maturity up to three years, 9-y Spread the maturity between six- and nine-years, and 12-y Spread the maturity greater than nine years. Either the US high-yield corporate bond index (HY) or the Moody's Aaa rated corporate bond spread relative to the 10-year Fed rate (Aaa) is used as an instrumental variable for spreads. Standard errors are robust to heteroskedasticity and within country serial correlation. ** significant at 5%; *** at 1%.

Table 14: First-Stage Regression of Spread: Inclusion of Zero-Coupon Bonds

Dependent Variable	3-y Spread	3-y Spread	9-y Spread	9-y Spread	12-y Spread	12-y Spread
HY	2.91***		2.86***		2.90***	
	[0.17]		[0.13]		[0.13]	
Aaa		2.41***		2.33***		2.33***
		[0.15]		[0.11]		[0.12]
Controls	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.20	0.20	0.31	0.28	0.24	0.21

Note: This table reports the first-stage regression of weekly series of spreads on an instrumental variable, controlling for real exchange rate, terms of trade, and investment grade dummy; differently from the main regression, zero-coupon bonds are now included into the sample. The number of observations is 6014. All variables are 6-month moving averages and demeaned for each country. Control variables are in logs. Spreads are in log-spreads $log(1 + r)$, and multiplied by one hundred. 3-y Spread is for the bond with the maturity up to three years, 9-y Spread the maturity between six- and nine-years, and 12-y Spread the maturity greater than nine years. Either the US high-yield corporate bond index (HY) or the Moody's Aaa rated corporate bond spread relative to the 10-year Fed rate (Aaa) is used as an instrumental variable for spreads. Standard errors are robust to heteroskedasticity and within country serial correlation. ** significant at 5%; *** at 1%.

C Full Model with Sudden Stop Shock and (Partial) Dilution

We present the full model with a sudden stop shock and dilution. The state space must be extended to include $s \in \{0,1\}$ an indicator for whether the country is in a sudden stop state. Under circumstances of (partial) dilution, the buyback bond price is a function of not only issuance characteristics but also the outstanding debt structure.

C.1 Value Functions

$$V\left(T,\delta,b,y,s\right) = \max_{d}\left\{V^{d}\left(y\right), \max_{x}\left\{V^{c}\left(T,\delta,b,y,s\right), V^{r}\left(T,\delta,b,y,s\right)\right\}\right\}$$

$$V^{d}\left(y\right) = u\left[h_{d}\left(y\right)\right] + \beta\,\mathbf{E}_{y'|y}\left\{(1-\psi)V^{d}\left(y'\right) + \psi V\left(0,0,0,y',0\right)\right\}$$

$$\begin{aligned} V^{c}\left(T,\delta,b,y,s\right) =& u\left[s h_{s}(y) + (1-s)y - (1+\delta)^{-T}\, b\right] \\ &+\beta\,\mathbf{E}_{y'|y,s'|s}\left\{\mathbb{1}_{T>0}\cdot V\left(T-1,\delta,b,y',s'\right) + \mathbb{1}_{T=0}\cdot V\left(0,0,0,y',s'\right)\right\} \end{aligned}$$

$$\begin{aligned} V^{r}\left(T,\delta,b,y,s\right) =& \max_{T',\delta',b'} u\left(c\right) + \beta\,\mathbf{E}_{y'|y,s'|s} V\left(T',\delta',b,y',s'\right) \\ \text{s.t. } c =& s\, h_{s}(y) + (1-s)y - (1+\delta)^{-T}\, b \\ & - q^{\text{bb}}\left(T-1,\delta,y,s,T',\delta',b'\right)\ b + q\left(T',\delta',b',y,s\right)\ b' \\ & q^{\text{bb}}\left(T-1,\delta,y,s,T',\delta',b'\right)\ b \geq q\left(T',\delta',b',y,s\right)\ b' \quad \text{if } s=1 \end{aligned}$$

C.2 Bond Prices

Risk-free bond price:

$$q^{\text{rf}}\left(T,\delta\right) = \frac{1}{R}\left\{(1+\delta)^{-T} + q^{\text{rf}}\left(T-1,\delta\right)\right\}$$

New issuance price:

$$\begin{aligned} q\left(T',\delta',b',y,s\right) =& \frac{1}{R}\,\mathbf{E}_{y'|y,s'|s}\left(1-d'\left(T',\delta',b',y',s'\right)\right)\left\{(1+\delta')^{-T'}\right. \\ &+ x\left(T',\delta',b',y',s'\right)\ q^{\text{bb}}\left(T'-1,\delta',y',s',T'',\delta'',b''\right) \\ &\left.+ \left(1-x\left(T',\delta',b',y',s'\right)\right)\ q\left(T'-1,\delta',b',y',s'\right)\right\} \end{aligned}$$

$$q^{\text{bb}}\left(T,\delta,y,s,T',\delta',b'\right) = q^{\text{rf}}\left(T,\delta\right)$$

Full dilution buyback price:

$$\begin{aligned} q^{\text{fd}}\left(T,\delta,y,s,T',\delta',b'\right) =& \frac{1}{R}\,\mathbf{E}_{y'|y,s'|s}\left(1-d'\left(T',\delta',b',y',s'\right)\right)\left\{(1+\delta)^{-T}\right. \\ &+ x\left(T',\delta',b',y',s'\right)\ q^{\text{fd}}\left(T-1,\delta,y',s',T'',\delta'',b''\right) \\ &\left.+ \left(1-x\left(T',\delta',b',y',s'\right)\right)\ q^{\text{fd}}\left(T-1,\delta,y',s',T'-1,\delta',b'\right)\right\} \end{aligned}$$

$\langle T'', \delta'', b'' \rangle$ are the optimal choices in state $\langle T', \delta', b', y', s' \rangle$, conditional on restructuring. Partial dilution buyback price:

$$q^{\text{pd}}\left(T, \delta, y, s, T', \delta', b'\right) = \xi\, q^{\text{rf}}\left(T, \delta\right) + \left(1 - \xi\right) q^{\text{fd}}\left(T, \delta, y, s, T', \delta', b'\right)$$

ξ controls the degree of dilution.