Bargaining over Taxes and Entitlements in the Era of Unequal Growth

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Abstract

Entitlement programs have become an increasing component of total government spending in the US over the last six decades. Using a political-economy model where parties bargain over taxes and entitlements, we argue that such dynamics can be explained by two factors. First, the country experienced a process of "unequal growth," where top earners became richer while the income level of the bottom 50 percent stagnated. This increased the demand by bottom earners to redistribute the additional resources. Second, institutional features (e.g., budget rules) affecting the determination of taxes and entitlements provide bargaining power to low income earners through a "status-quo effect." Using an infinite-horizon model calibrated to the US, we show that sustained bargaining power by a party representing the poor results in a rising share of entitlements consistent with the data. While entitlements programs are sub-optimally large, counter-factual experiments show that welfare outcomes are more equitable than those arising under alternative budget arrangements (e.g., bargaining only on taxes or only on entitlements).

Keywords: unequal growth, entitlements, inequality, redistribution, dynamic legislative bargaining, endogenous status quo, political economy, mandatory spending, budget rules.

JEL Classification: C7, D6, E6

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1 Introduction

Prior to the Great Depression, nearly all federal expenditures in the United States were discretionary. That is, spending did not occur in a given year unless Congress provided funding through an annual appropriations' bill. Following the Social Security Act of 1935, an increasing percentage of the federal budget became devoted to financing mandatory spending programs. This trend accelerated in the mid 1960s, and continued until the present day, as illustrated by Figure 1. The solid line represents the share of mandatory spending in total outlays (excluding debt interest payments), whereas the dotted line corresponds to discretionary spending.[1]

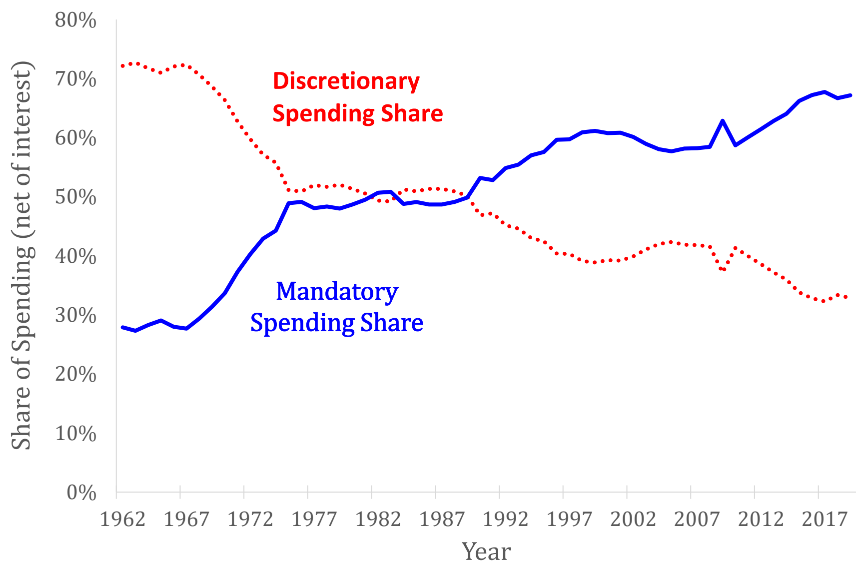


Figure 1: Share of mandatory and discretionary spending (in total spending), 1962-2019.

By 2019, mandatory spending represented over 60% of all government spending, as shown in the left panel of Figure 2. A key characteristic of these programs is that they need to be established under authorization laws and their generosity can only be modified with approval of the President and a majority (or super-majority) of members of Congress. The resulting bargaining process is similar to the one in which policymakers engage when attempting to modify the tax code.

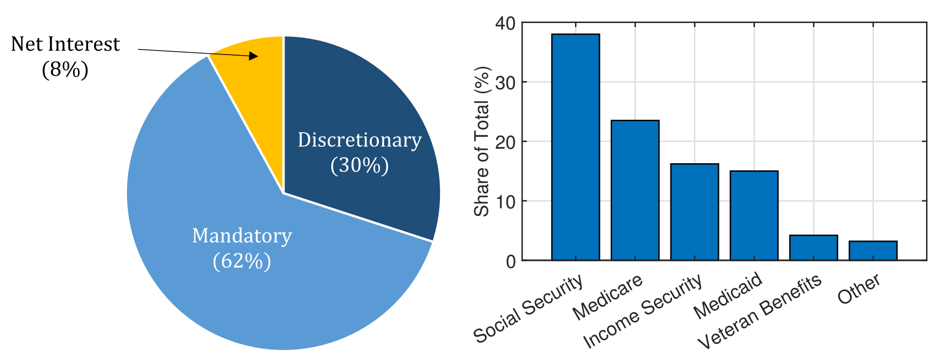


Figure 2: Composition of federal spending (left) and of mandatory spending (right), 2019

[1] Data sources are described in Section 7.1

While there is a wide range of goods financed by the government which are mandatory, the largest ones are *entitlement programs*, such as Social Security, Medicaid, Income Security, and Medicare (shown in the right panel of Figure 2), which provide private transfers to qualified individuals. These social welfare programs have specific criteria set by law, such as eligibility and benefit generosity, which generate significant redistribution towards eligible recipients from wealthier individuals.

The main objective of this paper is to explain the sustained expansion of entitlements programs since the 1960s. Our explanation relates this rise in entitlements to the process of *unequal growth* experienced by the US over the same period. Using Piketty et al. (2017) data, Figure 3 shows the evolution of pre-tax income for the average worker in the US (solid blue line) between 1962 and 2016, together with that of the bottom 50% of earners (dashed red line). While average income grew by 61% in real terms, the income of the bottom half of the population stayed relatively flat. This process of unequal growth, where the productivity gains only benefited the richest individuals in society, resulted in a significant increase in income inequality.

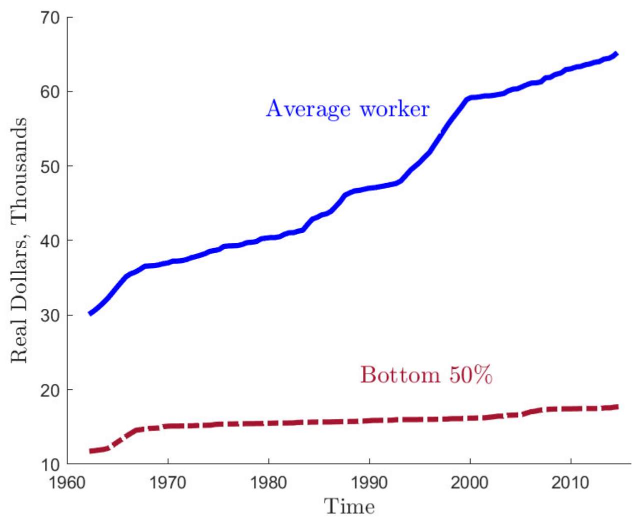


Figure 3: Pre-tax income in the US (in thousands of real dollars), 1962-2016.

We show that unequal growth can lead to an expansion of entitlements in a dynamic political-economy model where groups bargain over taxes and entitlements. Our argument has two parts. First, unequal growth results in higher demand for redistribution by bottom earners. Second, budget rules—composed by a *tax code* and *entitlement programs* in our benchmark model—, provide poor agents with the bargaining power needed to implement such redistribution. We show that both instruments, acting as "endogenous status-quo variables," are essential for this outcome. The pattern in Figure 1 cannot be replicated in an environment where groups bargain only over taxes or only over entitlements.

Our starting point is an endowment economy where two parties representing individuals from different income groups ("rich" and "poor") decide how to finance (pure) public goods and the degree of redistribution through entitlement programs. While both agents have the same preferences, they disagree on the degree of redistribution. This happens because rich agents are initially endowed with more resources than poor ones. As a result, the rich prefer a small government whose main role is to provide public goods, whereas the poor would like the government to provide generous entitlement programs, even if this involves high taxes. We do not restrict the set of instruments that policymakers have access to, so in principle the political equilibrium can be efficient. Our main friction, thus, is not distortionary taxation but instead disagreement regarding *where* on the Pareto frontier society should be. Under the "veil of ignorance" all agents agree on policies that decentralize the first best, which is both efficient and equitable. However, once they learn their type—at birth—each group prefers less equitable allocations that benefit them disproportionately. Political parties channel this through policy choices once they gain decision-making power. Their discretionary power to do so depends on the budgetary arrangements regulating how fiscal policy is determined.

We characterize a political equilibrium with bargaining over taxes and entitlements. We follow the protocol used in the bargaining literature under endogenous status quo to represent policy determination (see, for example, Bowen et al. (2014)). Every period, a party is selected at random to make a budget proposal involving public good spending, entitlement provision, and taxes. The proposer makes a take-it-or-leave-it offer to the opposition. The proposal is only implemented if the other party accepts. If the proposal is rejected, last period's taxes and entitlements (e.g., the status-quo) are implemented instead. What makes this environment different from one in which there are no budget rules is that taxes and entitlement programs require bipartisan consensus, and hence are harder to change.

We show that budget rules make fiscal policy more persistent, reducing the inefficiently high volatility of unconstrained environments. This is beneficial for welfare. In addition, these rules provide insurance against expropriation. By choosing the level of entitlements appropriately when in power, poor agents ensure a minimum degree of consumption through redistributive policies. By choosing taxes appropriately, rich agents shield themselves against excessively high taxes. In the long-run, however, budget rules result in allocations involving too much private consumption and under-provision of public goods.

In order to understand how unequal growth affects the share of entitlements over time, we conduct a quantitative experiment in which the level of income of rich individuals increases (permanently and unexpectedly), whereas the income of the poor stays the same. We calibrate the model to the US economy and impose a change in relative income between 1960 and 2010 consistent with Figure 3. To determine the proposer in every period, we use the time series of the US President's party during the same time horizon. For the purpose of our exercise, we map the Democratic party as the one representing the poor (pro-redistribution) and the Republican party as the on representing the rich (against "big governments"). Our simulation delivers an increase in the share of entitlements to total spending that is consistent with Figure 1. The increase in this share arises for two reasons. One of them is that Democrats have had proposal power more often than Republicans during our sample. A poor proposer always tries to increase entitlement programs, whereas a rich agent ties to reduce them. The second reason is that growth increases the size of the pie, and hence the fiscal capacity of the government. When these additional resources are unevenly distributed towards the rich, as implied by Figure 3, the set of policies that can arise in the bargaining model broadens. This happens because the rich would prefer to use some of the additional resources to provide pure public goods, which they are happy to fund through a tax increase. The poor, on the other hand, would like to increase taxes in order to expand the size of redistributive programs. Because consensus is needed to modify taxes and entitlements, there is room for negotiation: the poor agree to higher public good provision as long as this comes with larger private transfers.

A planner who faces an increase in resources endowed only to the rich would also choose to increase entitlement programs in order to redistribute these extra resources. However, and counter-factually, such change would happen instantaneously. In the bargaining equilibrium this change happens slowly over time, more in line with the US episode documented above. Through counterfactual experiments, we show that a political economy model in which policy decisions were discretionary, i.e, there is no status quo effect, would also be at odds with the data. In such environment, entitlements would jump between two extreme points every time there was a change in proposal power. Finally, we compared the evolution of entitlements in our benchmark model with one in which legislators only bargain over taxes or only over entitlements. We find that those fail to replicate the US experience. It is important, thus, that legislators bargain over both taxes and entitlements, in order to obtain an increase in the share of entitlements similar to the one in Figure 1.

Our model allows us to evaluate the welfare properties of alternative budget rules. We show that, for a fixed calibration of the model, budget rules on taxes and entitlements are significantly better for society than rules that would only affect one of these policies. Interestingly, low-income earners are significantly worse off when entitlements are mandatory but taxes are easy to change (e.g., when they are not status-quo variables). This happens because when taxes are easy to change, rather than bargained over, their status quo value is just large enough to cover for minimum spending requirements in equilibrium. This provides significant bargaining power to the rich. If tax cuts are easy to implement when in power, high-income earners can effectively keep the size of the government small and entitlement programs at bay. For completeness, we also consider a rule making public goods mandatory spending and show that, while such goods are more efficiently provided, the equilibrium is associated with welfare losses relative to our benchmark case. This is primarily due to the increased volatility of private consumption, which matters for our risk-averse agents.

Our paper makes three important contributions to the existing literature. First, we analyze the bargaining process over entitlement programs and taxes in an infinite horizon model with concave utilities. This allows us to evaluate how budget rules affect allocations in an environment where agents are heterogeneous in income levels and have preferences for smooth consumption profiles. A key finding is that introducing budget rules is not always beneficial to society. Our second contribution, is to point out that the type of fiscal policy targeted by a budget rule matters. This feature of budgetary design has been overlooked by most of the literature on dynamic bargaining. Our third, and main contribution, is to provide a rationale for the sustained increase in the share of entitlements in total spending over the last six decades. We show that a process of unequal growth, paired with long incumbency by the party representing the poor agents can result in such dynamics in a bargaining model with budget rules on taxes and entitlements. We see that as a novel contribution to existing literature. Finally, we also make a methodological contribution. Because we relax the linearity assumption on preferences used in most of the endogenous status-quo literature, characterization of the symmetric Markov-perfect equilibrium in the infinite-horizon dynamic game requires a numerical approach. We propose a numerical method that can robustly compute the equilibrium for a wide range of parameters. Computation is complex, because we have a multidimensional state space (e.g. two endogenous status-quo variables). Our method is inspired by Duggan and Kalandrakis (2012), and uses advances in the quantitative macroeconomics literature, such as those in Dvorkin et al. (2021). Matějka and McKay (2015) show that our multinominal logit discrete choice structure emerges endogenously from a setting with rational inattention, suggesting that one possible interpretation for these shocks in our model could be the difficulty of policymakers to fully inform themselves about the values associated with different fiscal policy proposals.

A brief literature review can be found in the next section. Section 3 defines the economic environment, while Section 4 characterizes theoretically efficient allocations for arbitrary Pareto weights and defines our concept of equity. In Section 5 we define the bargaining protocol and the political equilibrium. Section 6 characterizes a two-period model example to illustrate how these rules shape the equilibrium. The infinite-horizon dynamic model is solved quantitatively in Section 7 for a benchmark economy. Finally, Section 8 concludes and points venues for future research.

2 Literature Review

Our paper contributes to the literature studying redistributive policies and public good provision in the presence of political fictions. Similarly to Lizzeri and Persico (2001), we analyze how alternative arrangements can improve on allocations obtained under discretionary spending. While they focus on "winner-take-all" systems versus proportional systems, we consider budget rules in legislative models of bargaining instead. Our work is also related to the seminal papers by Lindbeck and Weibull (1987) and Meltzer and Richard (1981), who also consider redistribution through taxes, but in static models. Their work abstracts from bargaining, but instead study decisions implemented by representatives who are selected by voters. Meltzer and Richard (1981) shows that policies are determined by the preferences of the median voter, while Lindbeck and Weibull (1987) considers a probabilistic voting model, where policies maximize a weighed sum of the welfare of voters. In both environments, choices are unconstrained: there are no budget rules restricting policy choices.[2]

We model budget rules following the literature on legislative bargaining with endogenous status quo, along the lines of Baron and Ferejohn (1989). In our model, the tax code and the level of entitlements represent a status quo which remains in place unless some political group proposes an alternative allocation that is acceptable to the opposition. This mechanism creates a dynamic strategic link between the groups by impacting the trade-off faced by current policymakers and imposes limits on the degree of redistribution, in the spirit of Diermeier et al. (2017). Because fiscal policy directly impacts private allocations—and the size of the endowment is fixed—our work is related, more generally, to papers analyzing divide-the-dollar games with multilateral bargaining. One branch of this literature focuses on the continuous space (e.g., Kalandrakis (2004, 2010), Baron and Herron (2003), Anesi and Seidmann (2013), Nunnari (2021)), as in our theoretical analysis of the finite horizon model. The other branch restricts attention to choices in a discrete state space (e.g., Anesi (2010), Diermeier and Fong (2011), Diermeier et al. (2016), and Duggan and Kalandrakis (2012)), as we do in the numerical implementation. A thorough discussion of the recent developments in the legislative bargaining literature with endogenous status quo can be found in Eraslan et al. (2022). One important departure from these papers is that we consider government policies that affect both public and private goods. Moreover, we emphasize the distortions on public good provision arising from the bargaining process over private transfers.[3]

Our paper is also related to the literature determining the optimal provision of public goods in legislative bargaining models.[4] The closest paper to ours is Bowen et al. (2014), who analyze the welfare implications of mandatory spending rules on public goods. A key departure from their work is that we consider budget rules affecting private consumption allocations, through the determination of taxes and entitlements, where public good spending is discretionary. This, paired with initial income inequality, allows us to study how policies evolve in response to a process of unequal growth. Thus, our paper complements their findings by pointing out that the type of good targeted by the budget rule has important implications for its associated welfare gains, in an

[2]Section 7.4 develops this idea further.

[3]Battaglini and Coate (2007, 2008) do study legislative bargaining between private and public goods, but under the assumption of an exogenous status quo.

[4]Davila et al. (2009) study a cooperative bargaining approach to optimal public good provision.

environment where agents are risk averse. This finding is relevant because the largest mandatory spending programs in the United States are entitlements, which are mostly provided in the form of private transfers. There is an additional and more subtle difference between our paper and Bowen et al. (2014). A key underlying assumption in their model is the linearity in the utility of private goods. Because of linearity, fluctuations in private consumption or transfers resulting from bargaining do not generate welfare losses, level changes affect utility, but volatility does not. For us, because of their concave objective agents prefer smooth consumption profiles and therefore volatility in private consumption can generate significant welfare losses.[5]

Bouton et al. (2020) study the effect of introducing entitlement programs in an environment with public good provision, but using an alternating dictator approach. This delivers similar results to a bargaining environment in which all spending is discretionary and taxes are determined as a residual from the budget constraint of the government. They focus on the effects of entitlement programs on debt, from which we abstract (by imposing a balanced budget). Instead, we emphasize how the legislative bargaining process affects the evolution of the share of entitlements in the budget under unequal growth.

The discussion of rules versus discretion, as in, e.g., Amador et al. (2006), is also salient to our results. For example, Halac and Yared (2014) study the optimal level of discretion in fiscal policy when the economy faces persistent shocks. They show that when shocks are not i.i.d., an ex-ante optimal fiscal rule can create incentives for governments to accumulate maximal debt, becoming immiserated. Azzimonti et al. (2016) and Martin (2020) consider the welfare implications of balanced budget rules instead. We depart from these papers by considering mandatory rather than discretionary spending, but restricting the government ability to issue debt. Allowing for sovereign debt would be an interesting extension to our work. An excellent summary of the recent literature on budget rules in economies featuring debt in presidential systems can be found in Yared (2019). For recent work on the effect of budget rules on debt mitigation in a bargaining game see Piguillem and Riboni (2021).

Also relevant to our work is the literature studying the effects of power alternation on government policy, which includes Persson and Svensson (1989), Alesina and Tabellini (1990), Persson and Tabellini (2000), Acemoglu et al. (2011), and Azzimonti (2011), among others. These papers emphasize that political turnover introduces inefficiencies in a political equilibrium with no budget rules. We contribute to this literature by considering how budget rules can affect welfare in a model with legislative bargaining.[6] Considering concave utility functions over private consumption is key to our findings because individuals prefer smooth sequences of private and public consumption. This is an important departure from other papers studying the welfare consequences of budget rules.[7]

Our contribution is both qualitative and computational. Allowing for concave objectives embeds more macroeconomic realism to the setting but also raises technical challenges for the computation of optimal policies.[8] This is particularly the case when studying entitlement programs

[5]Moreover, when we replicate their model with concave utility on private consumption, we find that mandatory spending rules on public goods do not restore Pareto efficiency (they do involve Pareto improvements, though). The solution under mandatory spending on public goods with concave utilities is detailed in a previous version of this paper, Azzimonti et al. (2020).

[6]Agenda setting power is exogenously determined in our model. See Agranov et al. (2020) of an environment where it is endogenous instead.

[7]Bowen et al. (2017) discuss how concavity affects the desirability of mandatory spending rules, but also abstract from entitlements.

[8]There are few papers in the legislative bargaining models with endogenous status quo which introduce macroeconomic features. Piguillem and Riboni (2011) considers taxation as an endogenous status quo in the neoclassical growth model, whereas Grechyna (2021) considers endogenous resources.

because it calls for solving for a large number of value and policy functions over a multidimensional state space. We add to the computational bargaining literature by complementing the work of Duggan and Kalandrakis (2012), and to the macroeconomic literature by extending the techniques of Gordon (2019) and Chatterjee and Eyigungor (2020) to a political-economy environment with legislative bargaining.

3 Environment

Consider a discrete-time infinite horizon economy populated by two types of agents, R ("rich") and P ("poor"), of equal measure but different income levels, $y_R > y_P$. A draw of nature at the beginning of time determines the agent's type (with equal probability), and their type is fully persistent thereafter. Agents value private goods c and public goods g according to an additively separable instantaneous utility function

$$U(c, g) = u(c) + \theta u(g),$$

where u is strictly increasing and concave in both arguments, and satisfies $\lim_{x \to 0} u'(x) = \infty$. The constant $\theta > 0$ represents the relative importance of private to public goods in utility.

The government finances g with lump-sum taxes on the rich, denoted by τ_t, and on the poor, denoted by κ_t, and can redistribute income through an entitlement program, implemented as a cash transfer $\tilde{e}_t \geq 0$ to poor agents at each point in time. Denoting by e_t the net transfer that poor agents receive from the government, $e_t = \tilde{e}_t - \kappa_t$, the government budget constraint (GBC) satisfies

$$g_t + e_t \leq \tau_t. \tag{1}$$

The rich agents' consumption satisfies

$$c_{R,t} = y_R - \tau_t. \tag{2}$$

Whereas the poor's consumption is given by

$$c_{P,t} = y_P + e_t. \tag{3}$$

Note that whereas entitlements $\tilde{e}_t$ are positive, net transfers e_t need not be. We assume that there are constraints on the fiscal system ensuring a minimum level of private and public consumption, $c_{i,t} \geq \overline{x}$ for $i \in \{R, P\}$ and $g_t \geq \overline{x}_g$ with $\overline{x}$, $\overline{x}_g \geq 0$. We can interpret $\overline{x}$ as minimum consumption and $\overline{x}_g$ as the minimum amount of resources needed to run government operations and maintain law and order in society. Hence, spending on public goods takes this value unless a policymaker chooses to spend more. These constraints impose restrictions on the net transfers received by poor agents and the level of taxes paid by the rich,

$$e_t \geq \bar{x} - y_P, \qquad \tau_t \leq y_R - \bar{x} \quad \text{and} \quad \tau_t - e_t \geq \bar{x}_g. \tag{4}$$

For example, with $\bar{x} = \bar{x}_g = 0$, these bounds just restrict consumption of each agent to be non-negative. In such case, net transfers to the poor can be negative because entitlements are not enough to cover their taxes $e_t = \tilde{e}_t - \kappa_t < 0$. With $\bar{x} \geq y_P$, we capture an environment where the poor never pay taxes and net transfers are $e \geq 0$, so public goods are financed solely by the rich. In that case, we refer to e simply as entitlements. The values of $\bar{x}$ and $\bar{x}_g$, together with the degree of income inequality $y_R - y_P$, therefore, jointly determine the *fiscal capacity* of the government to finance public goods and the degree of redistribution that can be achieved.

Equations (1)-(3) imply that the total income in the economy $Y = y_R + y_P$ must be enough to cover private and public consumption levels, as stated in the resource constraint below.

$$c_{R,t} + c_{P,t} + g_t \leq Y. \tag{5}$$

With these, we can define allocations and evaluate lifetime utility as follows.

Definition 1. *An allocation* $\mathbf{a}$ *is a sequence of private and public goods,* $\mathbf{a} = \{c_{R,t}, c_{P,t}, g_t\}_{t=0}^{\infty}$. *These allocations induce lifetime welfare* $\mathcal{V}_i(\mathbf{a})$, *for each individual type* $i \in \{R, P\}$,

$$\mathcal{V}_i(\mathbf{a}) = \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, g_t),$$

given the discount factor $\beta \in (0, 1)$.

4 Efficient, Equitable, and Optimal Allocations

Before describing the political environment, it is useful to characterize the set of *Pareto Efficient* allocations.

Proposition 1 *The Pareto efficient allocations are time invariant and implicitly defined by*

$$Y - g^* = c_P^* + c_R^*$$
$$\theta\, u'(g^*) = \lambda u'(c_P^*) = (1 - \lambda) u'(c_R^*),$$

with $\lambda \in (0, 1)$ *denoting the Pareto-weight of poor agents.*

When utility is logarithmic,

$$g^* = \frac{\theta Y}{1 + \theta}, \quad c_P^* = \frac{\lambda Y}{1 + \theta}, \quad \textit{and} \quad c_R^* = \frac{(1 - \lambda) Y}{1 + \theta}. \tag{6}$$

Proof. See Appendix A.1. □

The efficient level of the public good satisfies the "Samuelson rule," which requires that the social marginal benefit of providing the public good (e.g. the sum of private marginal benefits) is equated to the social marginal cost. The solid blue line in Figure 4 illustrates the Pareto Frontier, corresponding to combinations of lifetime welfare given different values of $\lambda \in (0, 1)$. Note that the planner is not subject to the constraints on minimum private consumption $\bar{x}$.[9] We made this assumption precisely to emphasize that the concept of Pareto optimality may render solutions in which some agents are allocated infinitesimal levels of private consumption. In other words, we interpret $\bar{x}$ as a constitutional constraint limiting the set of allocations that society considers desirable, and its value is unrelated to efficiency considerations.[10]

It is standard in studies of optimal taxation to focus on Pareto efficient allocations in order to measure distortions from alternative fiscal policy plans. The underlying assumption is that if the policymaker had access to a complete set of instruments, she would be able to redistribute resources to achieve desirable societal outcomes. In other words, we typically work under the

[9]The constraint on g is non-binding in this environment, so it can be ignored.

[10]Adding this constraint shrinks the frontier in the direction of the optimal (efficient and equitable) allocation.

assumption that the Second Welfare Theorem holds and that all points in the Pareto frontier are, in principle, desirable.

The point of departure of this paper is different. We assume that policymakers have access to a complete set of fiscal policy instruments but that: (i) not all points in the frontier are desirable (as described above), and (ii) individuals disagree on the direction of policy once they know their type (i.e. at birth, in the first period). Note that in our economy, Pareto efficient allocations can be decentralized with time-invariant taxes and entitlements,

$$\tau^* = y_R - c_R^* \qquad \text{and} \qquad e^* = c_P^* - y_P.$$

These would be associated with some degree of redistribution, which, at the end of the day, depends on the value of the Pareto-weights. For example, with $\lambda = 0.5$, consumption would be equated $c_R^* = c_P^*$, resulting in significant redistribution from rich to poor agents. Clearly, R and P agents would disagree on value of λ that should be adopted by the government. Rich agents would like a policymaker with $\lambda \to 0$, whereas poor agents would prefer one with $\lambda \to 1$. Such extreme allocations would be efficient (e.g. at the Pareto Frontier), but they would not be equitable, as they would be associated with very different levels of welfare for both agents.

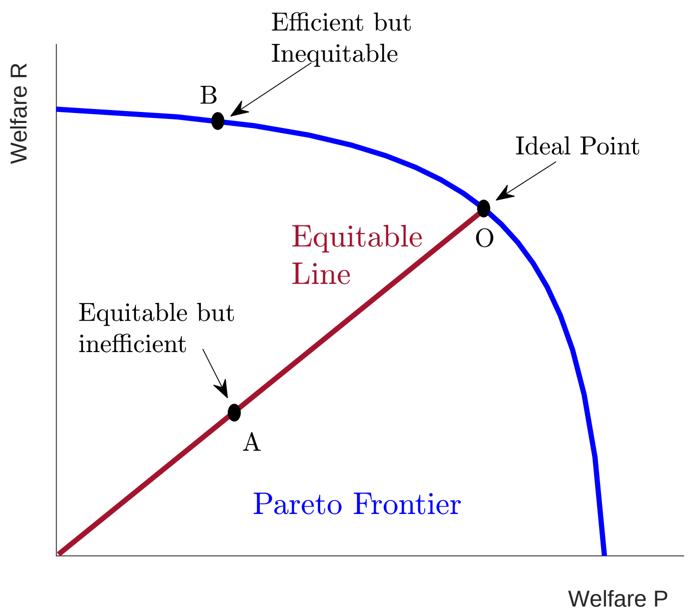


Figure 4: Efficient vs equitable allocations

We define an allocation $\mathbf{a}^e$ to be *equitable* when it is feasible and associated with the same level of lifetime welfare for all agents in society, $\mathcal{V}_R(\mathbf{a}^e) = \mathcal{V}_P(\mathbf{a}^e) \equiv \mathcal{V}(\mathbf{a}^e)$. These are illustrated in Figure 4 as points along the Equitable Line (solid red).

From the figure, we can see that there are allocations which are efficient but not equitable (such as B), while others are equitable but inefficient (such as A). What is, then, optimal from society's point of view? We call an allocation $\mathbf{a}^O$ *optimal* when it is both, efficient and equitable. In the plot, this corresponds to point O, where the Equitable Line intersects the Pareto Frontier. Optimal allocations can be found by simply setting $\lambda = 0.5$ in the expressions of Proposition 1.

When $u(x) = \ln(x)$ and $\theta = 1$, the optimal allocation prescribes that half of "the pie" Y is devoted to private goods and the rest is split evenly between the two agents,

$$g_t^o = \frac{Y}{2}, \quad \text{and} \quad c_{i,t}^o = \frac{Y}{4}. \tag{7}$$

Note that if agents were asked about their most preferred allocation under a veil of ignorance, i.e. before their type was revealed at birth, they would both agree on O being the ideal point. This is the case because we assumed that agents could be rich or poor with equal probability. Ex-post, on the other hand, they prefer to be closer to the extremes of the Pareto Frontier. This inconsistency in preferences, paired with an inability to commit to institutions ensuring an efficient and equitable allocation, is what gives raise to political parties in our model. Hence, our main friction is not distortionary taxation, but instead disagreement between individuals in society. The political equilibrium is discussed next.

5 Political Equilibrium

Because of income inequality, individuals from different income groups disagree over fiscal policy and, as a result, political parties naturally arise in this environment. We assume that there are two parties: R and P representing the interests of agents in each group. These parties bargain over policy each period and are subject to budget rules. More specifically, we assume that taxes and entitlements are governed by criteria set by enacted law. The latter corresponds to mandatory spending on transfers to the poor, directly affecting their level of private consumption. Unless a majority of legislators choose to change such laws, τ and e must take last period's values. We refer to them as the *tax code* and *entitlement programs*. On the other hand, public goods are considered to be discretionary spending. As such, there is no pre-determined value for g, which takes the minimum value $\bar{x}_g$ unless there is an explicit agreement by the two parties to spend more.

The protocol to change taxes and entitlements is similar to that in the legislative bargaining literature (see Baron and Ferejohn (1989)). A representative of one of the parties is selected at random to make a policy proposal, which consists of a triple $\{\tau, e, g\}$. The opposition party can accept or reject this proposal. If the proposal is accepted, it is implemented in the current period. If it is rejected, taxes and entitlements must take last period's values and $g = \bar{x}_g$. This implies that there are two relevant state variables, $\mathbf{s} = \{\bar{\tau}, \bar{e}\}$, as they determine status-quo values in case a proposal is rejected. The current proposer takes them as given when choosing the triple $\{\tau, e, g\}$.

To fix ideas, suppose that the proposer is of type P. Party R's choices are summarized by $d_R(\mathbf{s}) \in \{0, 1\}$, where $d_R(\mathbf{s}) = 1$ denotes that the proposal has been accepted. When a proposal is accepted, it becomes the new state, $\mathbf{s}' = \{\tau, e\}$. If the proposal is rejected, the status quo allocation from $\mathbf{s}$ is implemented, and next period's state remains the same, $\mathbf{s}' = \mathbf{s}$. There is a key dynamic component in this environment which was absent in the determination of Pareto efficient and optimal policies, the outcome of bargaining becomes the *endogenous status quo* next period.

5.1 Markov Perfect Equilibrium

Throughout our analysis, we assume that proposers alternate in power following a Markov process, where q denotes the probability of being the proposer tomorrow given that the party has proposal power today. We focus on a Markov Perfect Equilibrium. Under this equilibrium concept, policy functions only depend on payoff-relevant states of the economy, given by $\mathbf{s} = \{\bar{\tau}, \bar{e}\}$. Due to the existence of income inequality, equilibrium policy rules depend on the identity of the proposer. We denote by $\mathcal{G}_i(\mathbf{s})$ the MPE policy for public goods chosen by proposer i, whereas $\Psi_i(\mathbf{s})$ denotes the rule determining taxes imposed on the rich and $\mathcal{E}_i(\mathbf{s})$ the equilibrium policy determining net transfers to the poor. Consumption allocations of agent j determined by proposer i are denoted with $\mathcal{C}_{j,i}(\mathbf{s})$. The associated continuation utilities are $V_i(\mathbf{s}')$ if the proposer remains the proposer next period and $W_i(\mathbf{s}')$ if out of power.

Suppose that P is the current proposer, her maximization problem can be written as

$$\max_{\{\tau,e,g\}} u(c_P) + \theta u(g) + \beta \Big\{ qV_P(\pi_P) + (1-q)W_P(\pi_P) \Big\} \tag{8}$$

where we used the fact that on the equilibrium path the proposal is accepted, $\mathbf{s}' = \{\tau, e\} \equiv \pi_P$. The constraints are eqs. (1)-(4), and the *acceptance constraint*

$$\begin{aligned} u(c_R) + \theta u(g) + \beta \Big\{ (1-q)V_R(\pi_P) + qW_R(\pi_P) \Big\} \quad &\geq \\ u(y_R - \bar{\tau}) + \theta u(\bar{x}_g) + \beta \Big\{ (1-q)V_R(s) + qW_R(s) \Big\} & \end{aligned} \tag{9}$$

The bottom part of the equation denotes the dynamic payoff to the opposition party when the proposal is rejected, that is, the payoff from keeping the status quo. Recall that when no agreement is reached, $g = \bar{x}_g$ and $c_R = y_R - \bar{\tau}$. The acceptance constraint ensures that the proposal is accepted if and only if the payoff from the proposal is individually rational for the respondent, i.e., it provides a payoff that is at least as high as the payoff under the status quo s. The expression makes it clear that the budget rule defines a lower bound for the welfare level attained by the opposition R. It also makes it clear that the lower bound $\bar{x}_g \geq 0$ controls how binding equation (9) is at every point in time. When $\bar{x}_g$ is very small, rejecting a proposal can be very costly for the opposition, so larger deviations from the status quo will become attainable for the proposer.

The acceptance constraint of party P when R is in power is similarly defined,

$$\begin{aligned} u(c_P) + \theta u(g) + \beta \Big\{ (1-q)V_P(\pi_R) + qW_P(\pi_R) \Big\} \quad &\geq \\ u(y_P + \bar{e}) + \theta u(\bar{x}_g) + \beta \Big\{ (1-q)V_P(s) + qW_P(s) \Big\} & \end{aligned}$$

since $c_P = y_P + \bar{e}$ when the proposal is rejected.

Finally, we have that in the MPE, the value function of a type-P proposer satisfies

$$V_P(\mathbf{s}) = u\Big(\mathcal{C}_{P,P}(\mathbf{s})\Big) + \theta u\Big(\mathcal{G}_P(\mathbf{s})\Big) + \beta \Big\{ qV_P\Big(\Pi_P(\mathbf{s})\Big) + (1-q)W_P\Big(\Pi_P(\mathbf{s})\Big) \Big\} \tag{10}$$

with next period's status quo given by today's equilibrium choices by proposer P, namely $\Pi_P(\mathbf{s}) = \{\Psi_P(\mathbf{s}), \mathcal{E}_P(\mathbf{s})\}$. The value function of type P when out of power satisfies

$$W_P(\mathbf{s}) = u\Big(\mathcal{C}_{P,R}(\mathbf{s})\Big) + \theta u\Big(\mathcal{G}_R(\mathbf{s})\Big) + \beta \Big\{ (1-q)V_P\Big(\Pi_R(\mathbf{s})\Big) + qW_P\Big(\Pi_R(\mathbf{s})\Big) \Big\} \tag{11}$$

as policies are chosen by party R in such case, with $\Pi_R(\mathbf{s}) = \{\Psi_R(\mathbf{s}), \mathcal{E}_R(\mathbf{s})\}$. We can now formally define the Markov perfect equilibrium of this game.

Definition 2. *A MPE with legislative bargaining is a set of value functions $\{V_i(\mathbf{s}), W_i(\mathbf{s})\}$, policy functions $\Pi_i(\mathbf{s}) = \{\Psi_i(\mathbf{s}), \mathcal{E}_i(\mathbf{s})\}$, allocations $\Big\{\mathcal{C}_{j,i}(\mathbf{s}), \mathcal{G}_i(\mathbf{s})\Big\}$, and acceptance rules $d_j(\mathbf{s})$ for proposer i and opposition $j \neq i$ where $i, j \in \{R, P\}$, such that*

- *Proposer i chooses public good g and fiscal policy $\pi_i = \{\tau, e\}$ to maximize problem (8) subject to the budget constraints eqs. (1)-(3), the bounds eq. (4), and the acceptance constraint, eq. (9). Given the value functions $V_i(\mathbf{s})$ and $W_i(\mathbf{s})$, the acceptance decision $d_j(\mathbf{s})$, and the rules chosen by the opposition party j, $\Pi_j(\mathbf{s})$, these define the policy functions $\Pi_i(\mathbf{s})$. The problem of proposer j is analogously defined.*
- *Given the policy functions $\Pi_j(\mathbf{s})$ and $\Pi_i(\mathbf{s})$, the value functions $V_i(\mathbf{s})$ and $W_i(\mathbf{s})$ satisfy equations (10) and (11), respectively. The value functions $V_j(\mathbf{s})$ and $W_j(\mathbf{s})$ are analogously defined.*
- *Given $V_i(\mathbf{s})$ and $W_i(\mathbf{s})$, for any proposal π_i and status quo $\mathbf{s}$, the acceptance strategy $d_j(\mathbf{s}) = 1$ if and only if eq. (9) holds. The acceptance rule $d_i(\mathbf{s})$ is analogously defined.*

The first condition states that policy rules are the ones that solve the problem of the proposer, given continuation utilities and an acceptance rule for the opposition party. The second condition defines value functions as a fixed point using policy functions under an accepted proposal. The last condition determines that the opposition party accepts the proposal whenever its welfare is at least as large as under the status quo.

5.2 Discretionary Budget Arrangement

Before characterizing the equilibrium with budget rules on τ and e, it is useful to briefly discuss the proposer's optimal choices in an unconstrained environment (e.g., one with no budget rues). There is no pre-determined value for expenditures in public goods or entitlements in the tax system, other than those that guarantee minimum consumption levels. Hence, if a policy proposal is rejected, spending on public goods takes the minimum value $\bar{x}_g$ and entitlements equal the lower bound $e = \bar{x} - y_P$. This is typically defined as "discretionary spending" by the US Office of Management and Budget. Without budget rules (and hence no tax code), taxes are determined residually from the government's budget constraint, so $\tau = \bar{x}_g + \bar{x} - y_P$. That is, taxes should be enough to cover the minimum provision of public goods and entitlements. The optimization problem is the same as before, but without imposing the acceptance constraints (9).

The solution to this problem is analogous to the ones in the citizen candidate models of Osborne and Slivinski (1996) or Besley and Coate (1997), also known as "alternating dictator" models. Because total income is constant over time and the government is subject to a balanced budget, there is no dynamic state variable. Therefore, the problem of a proposer choosing policies not subject to budget rules is static, with continuation utility being irrelevant. Proposer i sets

$$\theta u'(g^D) = u'(c^D_{i,i}), \tag{12}$$

equating the marginal benefit of public good provision to her private marginal cost. Proposition 2 further characterizes the solution for the tractable case of $\theta = 1$ and we relegate the more cumbersome general case for Section 2 of the Online Appendix.

Proposition 2 *Under logarithmic utility, $\theta = 1$, $\bar{x} = \bar{x}_g \equiv \bar{x}$, and absent budget rules, fiscal policy satisfies*

$$P \textit{ in power:} \quad \tau^D_P = y_R - \bar{x} \quad \textit{and} \quad e^D_P = \frac{\Delta - \bar{x}}{2}.$$

$$R \textit{ in power:} \quad \tau^D_R = \frac{\Delta + \bar{x}}{2} \quad \textit{and} \quad e^D_R = \bar{x} - y_P,$$

where $\Delta = y_R - y_P$ denotes income inequality. The resulting allocations when i is the incumbent are given by

$$g^D = \frac{Y - \bar{x}}{2}, \quad c^D_{i,i} = g^D, \quad \textit{and} \quad c^D_{j,i} = \bar{x} \quad \textit{for } j \neq i.$$

Proof. See Appendix A.2. □

The proposer expropriates the other group as much as possible, providing them with the minimum feasible level of consumption when in power. When P is the proposer, taxes on the rich are maximal, $\tau_P^D = y_R - \bar{x}$ whereas their consumption hits the lower bound $c^D_{R,P} = \bar{x}$. The remaining of the budget is divided between the public good and the consumption of the poor. When private and public goods have the same weight, $\theta = 1$, the budget is split evenly, $c^D_{P,P} = g^D$. When R is the proposer, net transfers to the poor are minimal, so they consume at $c^D_{P,R} = \bar{x}$. Note that while the two parties disagree on the burden of taxation, they both choose the same provision of public goods, g^D and policies that result in the same total level of private consumption $c^D = c^D_{R,i} + c^D_{P,i}$. Hence, public good provision and total consumption do not depend on the identity of the party in power.

Is this solution Pareto efficient? No. To see this, assume both types of agents have the same Pareto-weight $\lambda = \frac{1}{2}$ (in this case, $c^D > c^*$ for sure). There are two sources of Pareto inefficiency when the proposer is not subject to budget rules and there is power alternation, $q < 1$. The first one is a static inefficiency which arises because, to the extent that $\bar{x} > 0$, the incumbent under-provides public goods relative to the planner,

$$g^D < g^* \quad \text{and} \quad c^D > c^*.$$

The solution when $\overline{x} > 0$ never satisfies the Samuelson rule because the incumbent equates the marginal cost of public goods to her private marginal benefit, whereas the planner would equate it to the social marginal benefit. In other words, incumbent i ignores the welfare gains to group j of providing g and this results in over-provision of private goods and under-provision of public goods, regardless of the identity of the incumbent.

The second source of inefficiency is dynamic and arises because private consumption fluctuates between $\frac{Y-\bar{x}}{2}$ and $\bar{x}$ with the identity of the incumbent, whereas the efficient solution is constant. Curvature in the utility function of agents implies that they would prefer a smooth consumption sequence to a volatile one. In a political equilibrium without rules, the volatility induced by power alternation reduces lifetime utility, and it is a dynamic source of inefficiency in this model. Allowing for power alternation, however, does have a benefit to society: By changing the decision-maker every period, the political system enables more equitable allocations. Without it, one type would always consume the lower bound $\bar{x}$.

6 Two-Period Model

The easiest way to understand the effect of budget rules on allocations is through the analysis of a two-period version of the model, where analytical results are feasible. The infinite-horizon dynamic model is studied in Section 7.1. To find the solution to the Markov Perfect Equilibrium, we solve the problem backwards, starting from the second period.[11]

6.1 Second-Period Characterization.

The second-period proposer takes the status quo $\mathbf{s} = \{\bar{\tau}, \bar{e}\}$ as given. Because the economy ends this period, there is no continuation utility. The analysis allows us to understand how the status

[11]Given that this is a two-person, two-period, complete information extensive form game, we focus on its unique Subgame Perfect Equilibrium (SPE). Second-period strategies depend on histories only through the status quo.

quo affects the choice set and the relative bargaining power of the two groups, while ignoring the dynamic consequences of this choice.

Suppose that party P has proposal power this period. The incumbent proposes $\{g_2, \tau_2, e_2\}$, given status quo $\boldsymbol{s}$, in order to maximize her static payoff,

$$\max_{\{\tau_2, e_2, g_2\}} u(c_{P,2}) + \theta u(g_2) \quad \text{s.t.}$$

$$u(c_{R,2}) + \theta u(g_2) \geq u(y_R - \bar{\tau}) + \theta u(\bar{x}_g), \tag{13}$$

$$\text{and eqs. (1)-(4),}$$

The acceptance constraint (13) ensures that the proposal is accepted if and only if the payoff from the proposal weakly exceeds the payoff under the status quo $\boldsymbol{s}$. In the analysis that follows, we assume that the utility is logarithmic, $\theta = 1$, and $\bar{x} = \bar{x}_g \equiv \bar{x}$, as it greatly facilitates exposition.

When the acceptance constraint is not binding, the solution is akin to that under no budget rules, characterized in Proposition 2. This solution corresponds to the unconstrained best from P's point of view. When inequality (13) binds, the solution is more involved and depends on the status-quo value of taxes on the rich, $\bar{\tau}$. The full result is characterized in the following proposition.

Proposition 3 *Let utility be logarithmic $u(.) = \ln(.)$, $\theta = 1$, and $x_g = \bar{x}$. In the last period, the unique equilibrium proposal for proposer P satisfies:*

$$\mathcal{E}_{P,2}(\boldsymbol{s}) = \begin{cases} \frac{\Delta}{2} - \frac{2\bar{x}[y_R - \bar{\tau}]}{Y}, & \text{if} \quad \bar{\tau} < \frac{\Delta}{2} \\ \bar{\tau} - \bar{x}, & \text{if} \quad \bar{\tau} \in \left[\frac{\Delta}{2}, \tau_R^D\right) \\ e_P^D, & \text{if} \quad \bar{\tau} \geq \tau_R^D. \end{cases} \qquad \Psi_{P,2}(\boldsymbol{s}) = \begin{cases} y_R - \frac{2\bar{x}[y_R - \bar{\tau}]}{Y}, & \text{if} \quad \bar{\tau} < \frac{\Delta}{2} \\ \tau_P^D, & \text{if} \quad \bar{\tau} \in \left[\frac{\Delta}{2}, \tau_R^D\right) \\ \tau_P^D, & \text{if} \quad \bar{\tau} \geq \tau_R^D. \end{cases}$$

and

$$\mathcal{G}_{P,2}(\boldsymbol{s}) = \begin{cases} g^*, & \text{if} \quad \bar{\tau} < \frac{\Delta}{2} \\ y_R - \bar{\tau}, & \text{if} \quad \bar{\tau} \in \left[\frac{\Delta}{2}, \tau_R^D\right) \\ g^D, & \text{if} \quad \bar{\tau} \geq \tau_R^D. \end{cases}$$

The associated private consumption allocations are

$$\mathcal{C}_{P,P,2}(\boldsymbol{s}) = y_P + \mathcal{E}_{P,2}(\boldsymbol{s}) \quad \textit{and} \quad \mathcal{C}_{R,P,2}(\boldsymbol{s}) = y_R - \Psi_{P,2}(\boldsymbol{s}).$$

Proof. See Appendix A.3. □

The only relevant state variable for proposer P is the status quo level of taxes on the rich, $\bar{\tau}$. This is the case because the rich use their veto power to block policy changes that deliver welfare levels below those obtained under the status quo. The effect of alternative $\bar{\tau}$ values is illustrated in Figure 5 for a numerical example with $Y_R = 1.3$, $Y_P = 0.1$, and $\bar{x} = 0.1$, where we plot equilibrium policies (top panel) and allocations (bottom panel) as functions of $\bar{\tau}$.[12] When $\bar{\tau} \geq \tau_R^D$, status quo taxes on the rich are so large that the proposer is able to implement the unconstrained

[12]The magnitude of $\bar{x}$ matters for the relevance of budget rules. The closer $\bar{x}$ is to the optimal value $g^o = \frac{Y}{2}$, the less important is the political bargaining process. This is because when $\bar{x} = \frac{Y}{2}$, the bounds on taxes and entitlements imply that the only feasible allocations are the optimal ones, so policy decisions become trivial.

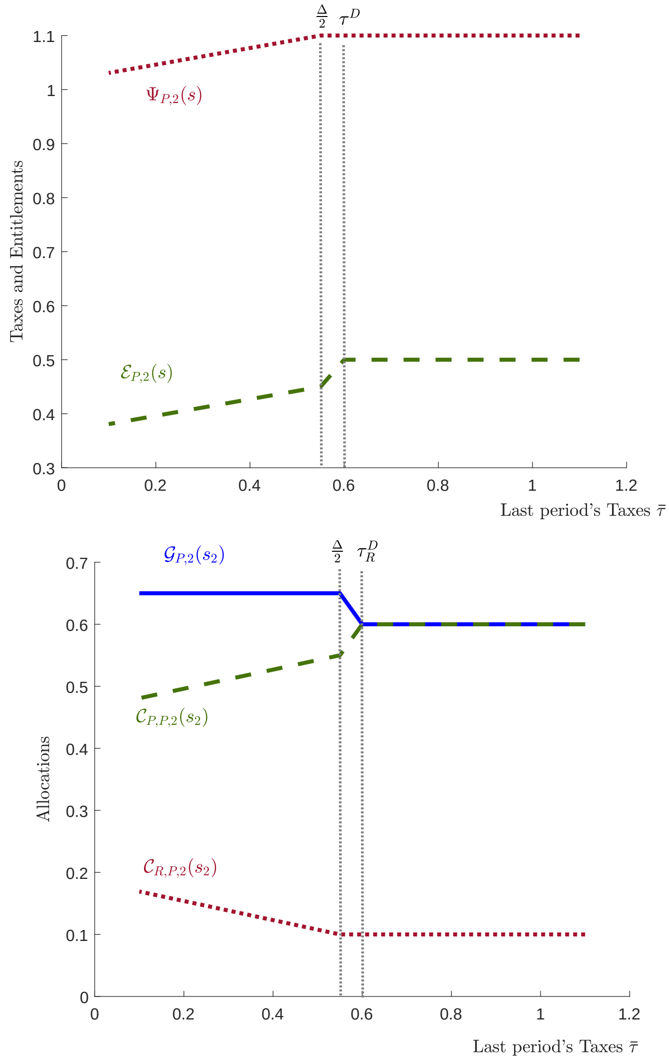


Figure 5: Second period policies and allocations under proposer P. Parameters: $y_R = 1.3$, $y_P = 0.1$, and $\overline{x} = 0.1$

solution, τ_P^D and e_P^D. Because the acceptance constraint is not binding, the proposer just equates her marginal utility of private consumption to that of public consumption, implying $\mathcal{G}_{P,2}(\boldsymbol{s}) = g^D$ and $\mathcal{C}_{P,P,2}(\boldsymbol{s}) = c_{P,P}^D$.

When intermediate taxes are established in the tax code, $\bar{\tau} \in \left[\frac{\Delta}{2}, \tau_R^D\right)$, proposing the unconstrained policies is no longer acceptable for the opposition. Group R is better off rejecting the proposal and keeping taxes and entitlements at their status quo values. Anticipating this, proposer P offers an alternative mix that guarantees the opposition's minimum welfare under the status quo, so that constraint (13) becomes binding, but that make P slightly better off. In order to induce the opposition to accept the proposal, proposer P needs to either impose lower taxes (e.g. increase $c_{R,2}$ above the minimum $\bar{x}$) or provide more public goods at the expense of her own consumption (through lower entitlements). Given that P enjoys consuming public goods (but derives no utility from the opposition's private consumption), it is best to offer τ_P^D and instead reduce entitlements below e_P^D. This results in higher provision of public goods and a slightly lower c_P

than under discretionary spending (e.g. no budget rules environment). The rich are willing to accept this proposal even though their consumption is set at the lower bound. When taxes are below $\frac{\Delta}{2}$, the opposition has so much bargaining power, that proposer P is forced to reduce taxes and entitlements even further. The only proposal that would give P high consumption involves $g_2 = g^*$, the Samuelson level of the public good provision. Interestingly, consumption inequality is minimal when taxes on the rich are low, despite the built in mechanism to have more equality through entitlement programs. This is the case because low taxes give more bargaining power to rich agents when P is in power, limiting the poor's ability to expropriate the rich.

For completeness, it is useful to characterize the policy rules that would be chosen by proposer type R if in power in the second period under the assumptions of Proposition 3.

Proposition 4 *Let utility be logarithmic $u(.) = \ln(.)$, $\theta = 1$, and $\bar{x} = \bar{x}_g \equiv \bar{x}$. In the last period, the unique equilibrium proposal for proposer R satisfies:*

$$\mathcal{E}_{R,2}(\boldsymbol{s}) = \begin{cases} e_R^D, & \text{if} \quad \bar{e} < e_P^D \\ e_R^D, & \text{if} \quad \bar{e} \in \left[e_P^D, \frac{\Delta}{2}\right) \\ \frac{2\bar{x}[y_P - \bar{e}]}{Y} - y_P, & \text{if} \quad \bar{e} \geq \frac{\Delta}{2}. \end{cases} \qquad \Psi_{R,2}(\boldsymbol{s}) = \begin{cases} \tau_R^D, & \text{if} \quad \bar{e} < e_P^D \\ \bar{x} - \bar{e}, & \text{if} \quad \bar{e} \in \left[e_P^D, \frac{\Delta}{2}\right) \\ \frac{\Delta}{2} + \frac{2\bar{x}[y_P - \bar{e}]}{Y} & \text{if} \quad \bar{e} \geq \frac{\Delta}{2}, \end{cases}$$

$$\mathcal{G}_{R,2}(\boldsymbol{s}) = \begin{cases} g^D, & \text{if} \quad \bar{e} < e_P^D \\ y_P + \bar{e}, & \text{if} \quad \bar{e} \in \left[e_P^D, \frac{\Delta}{2}\right) \\ g^*, & \text{if} \quad \bar{e} \geq \frac{\Delta}{2}. \end{cases}$$

with associated consumption $\mathcal{C}_{P,R,2}(\boldsymbol{s}) = y_P + \mathcal{E}_{R,2}(\boldsymbol{s})$ and $\mathcal{C}_{R,R,2}(\boldsymbol{s}) = y_R - \Psi_{R,2}(\boldsymbol{s})$.

Proof. See Appendix A.4. □

For an R proposer, the relevant thresholds are determined by the status quo level of net transfers to the poor $\bar{e}$. Under the assumption that $\bar{x} = y_P$, these are just thresholds on entitlement levels. The intuition behind the solution above is analogous to the one described before. When existing law establishes a low level of entitlements, a rich proposer chooses her best unconstrained choice (e.g. the one under no budget rules). As $\bar{e}$ exceeds the first threshold (but not the second), the acceptance constraint of the poor becomes binding, so R finds it optimal to propose a policy combo that results in higher public good provision but lower consumption to herself, so as to keep entitlements as low as possible. Once $\bar{e} \geq \frac{\Delta}{2}$, the best proposal involves the Samuelson level of public good provision g^* and the smallest possible entitlement level and taxes that would induce P to accept the proposal. The highest consumption equality is achieved when $\bar{e}$ is large under a proposer of type R in the last period of the finite horizon game.

Summarizing, the second period choices depend on status quo values of entitlements and taxes. Low existing taxes force poor proposers to provide private consumption allocations which are more equitable and public goods which are closer to the first best. This is because under low $\bar{\tau}$, the opposition has high bargaining power and can veto allocations that would expropriate their type's income significantly in order to generate redistribution. On the other hand, when R is the proposer, higher status quo values of entitlements achieve more equitable allocations instead. By having a relatively high minimum consumption to the poor (as established by existing laws), the rich can only make themselves better off by providing public goods above the unconstrained levels. The reason being that high tax cuts can be easily be vetoed by poor agents when $\bar{e}$ is large. In other words, *entitlement laws protect the poor against policies preferred by the rich whereas a tax code* *protects the rich against excessive taxation preferred by the poor.* Discretionary spending, in the form of public goods, in our model, serves as a bargaining chip for a proposer to change the status quo in her favor. This is accepted by the opposition because both types of agents value public goods.

One important remark is in place before we move to the first period. That only one of the elements in $\boldsymbol{s}$ is relevant for each policymaker is not a general result. It only holds in the last period of a finite horizon game when there is no continuation utility. In the infinite horizon model, on the other hand, both status quo values are going to be payoff relevant. This is discussed at length in Section 7.2, where we illustrate how policy rules depend on both states.

6.2 First-Period Characterization.

We now characterize first-period allocations and ask whether an unconstrained P-proposer (e.g. whose acceptance constraint is slack) would find it beneficial to choose g^D and corresponding consumption allocations or deviate from them– knowing her choices in the first-period will impact her continuation value in the second-period by changing the status quo—. Building on our previous results, we work under the assumption that $\theta = 1$ and utility is logarithmic.

A proposer facing $q = 1$ would find it optimal to choose the unconstrained policies in both periods. There is no gain to introduce a tax code or an entitlement program that would move allocations away from g^D and $c^D_{P,P}$, as these achieve the highest level of welfare for the proposer at every point in time. This is no longer the case under uncertainty, as the appropriate choice of taxes and entitlements could 'tie the hands' of her successor. By choosing alternative taxes and entitlements today, a proposer can manipulate future decisions through the endogenous status quo channel. More importantly, altering the optimal policy mix today can provide insurance against expropriation in the event that the opposition gains proposal power.

To show this, consider a situation where P proposes a public good provision allocation g_1 and policies $\pi_{P,1} = \{\tau_1, e_1\}$. The acceptance constraint is slack today (by assumption), but the proposer understands that these policies become the status quo next period, $\boldsymbol{s} = \pi_{P,1}$. Her maximization problem is

$$\max_{\{\tau_1, e_1, g_1\}} \quad \ln(c_{P,1}) + \ln(g_1) + \beta \{ q V_P(\pi_{P,1}) + (1-q) W_P(\pi_{P,1}) \} \tag{14}$$

$$\text{s.t. eqs. (1) and (3)} - (4),$$

where we have used the fact that $\boldsymbol{s} = \pi_{P,1}$. The value functions $V_P(\pi_{P,1})$ and $W_P(\pi_{P,1})$ can be obtained by evaluating the solution characterized in Propositions 3 and 4 into the utility in the second period. The continuation utility of proposer P if she stays in power tomorrow is given by

$$V_P(\tau_1) = \begin{cases} \ln(g^*) + \ln\left(\frac{Y^2 - 4\bar{x}[y_R - \tau_1]}{2Y}\right), & \text{if} \quad \tau_1 < \frac{\Delta}{2}. \\ \ln(y_R - \tau_1) + \ln(y_P + \tau_1 - \bar{x}), & \text{if} \quad \tau_1 \in \left[\frac{\Delta}{2}, \tau_R^D\right) \\ \ln(g^D) + \ln\left(c^D_{P,P}\right), & \text{if} \quad \tau_1 \geq \tau_R^D. \end{cases}$$

This is because only τ_1, the level of entitlements received by the opposition, may constrain future decisions when P remains in power. If R becomes next period's proposer, then it is the current entitlement level e_1 what will constrain R's decisions instead. The continuation utility for P in such case would be

$$W_P(e_1) = \begin{cases} \ln(\bar{x}) + \ln(g^D), & \text{if} \quad e_1 < e_P^D \\ \ln(\bar{x}) + \ln(y_P + e_1), & \text{if} \quad e_1 \in \left[e_P^D, \frac{\Delta}{2}\right) \\ \ln\left(\frac{2\bar{x}[y_P+e_1]}{Y}\right) + \ln\left(g^*\right), & \text{if} \quad e_1 \geq \frac{\Delta}{2}. \end{cases}$$

This function is computed by replacing R's optimal choices on P's utility next period.

Proposer P chooses allocations in the first period to maximize eq. (14), which can be re-written as

$$\max_{\{\tau_1, e_1\}} \ \ln(y_P + e_1) + \ln(\tau_1 - e_1) + \beta\left\{qV_P(\tau_1) + (1-q)W_P(e_1)\right\},$$

subject to the lower bound constraints. We have used eqs. (1)-(3) to write down all the allocations in the first period in terms of policy.

Inspection of the problem above reveals that it is optimal for proposer P to set current taxes as high as possible, $\tau_1 = y_R - \bar{x}$. This maximizes resources in the current period and improves P's bargaining power tomorrow. The first order condition with respect to e_1 is

$$\underbrace{\frac{1}{c_{P,1}}}_{MU_c} - \underbrace{\frac{1}{g_{P,1}}}_{MU_g} = \underbrace{\beta(1-q)\frac{\partial W_P(e_1)}{\partial e_1}}_{\text{wedge}_c > 0}. \tag{15}$$

In the absence of uncertainty (e.g. $q = 1$), the proposer would set the left hand side of the equation to zero, which achieves the unconstrained (e.g. no budget rules) solution. At that point, the private marginal costs and benefits of entitlements are equated. When $q < 1$, the proposer finds it optimal to distort the solution because, by choosing $e_1 > e_P^D$ it is possible to affect the status quo inherited by the opposition if group R becomes the proposer next period, therefore increasing her own welfare, $W_P(e_1)$, in that state of the world. The solution to the first period allocations under budget rules is characterized in Proposition 5.

Proposition 5 *Suppose that the acceptance constraint is not binding in $t = 1$. The unique proposal strategy for proposer P under budget rules is:*

$$\mathcal{E}_{P,1} = \frac{2e_P^D + \beta(1-q)\tau_P^D}{2 + \beta(1-q)}, \qquad \Psi_{P,1} = \tau_P^D, \quad \textit{and} \qquad \mathcal{G}_{P,1} = \frac{2}{2+\beta(1-q)}g^D,$$

with associated allocations

$$\mathcal{C}_{P,P,1} = \frac{2\Big(1+\beta(1-q)\Big)}{2+\beta(1-q)}c_{P,P}^D \qquad \textit{and} \qquad \mathcal{C}_{R,P,1} = \bar{x}.$$

Proof. See Appendix A.5 □

The solution above determines $\boldsymbol{s}$, and hence the constraint faced by tomorrow's policymaker. Her preferred level of entitlements depends on the probability q. Under uncertainty, the proposer sets $\mathcal{E}_{P,1} > e_P^D$, understanding that this results in too little public good provision today and too much private consumption. This is a current cost because it distorts the allocation relative to the unconstrained case. The gain arises in the future: by establishing an overly generous entitlement program today, it forces the opposition to offer a better policy mix tomorrow. By over-spending on entitlement programs, the current proposer ensures a better bargaining position next period.

It is easy to show that if R was the first-period proposer, she would choose taxes and entitlements to favor her own private consumption while sacrificing the provision of public goods. In particular, she would set entitlements to their lowest possible level $\mathcal{E}_{R,1} = e_R^D$, in order to ensure maximum bargaining power in case she remains the proposer in the second period. She would adjust taxes trading off current distortions against insurance against future expropriation in response to alternative values of q. The resulting allocations are summarized in Proposition 6.

Proposition 6 *Suppose that the acceptance constraint is not binding in $t = 1$. The unique proposal strategy for proposer R under an entitlement rule is:*

$$\mathcal{E}_{R,1} = \overline{x} - y_P, \qquad \Psi_{R,1} = \frac{y_R + (1+\beta\,(1-q))\, e_R^D}{2+\beta\,(1-q)}, \quad \textit{and} \qquad \mathcal{G}_{R,1} = \frac{2}{2+\beta(1-q)} g^D,$$

with associated allocations

$$\mathcal{C}_{R,R,1} = \frac{2\Big(1+\beta(1-q)\Big)}{2+\beta(1-q)} c_{R,R}^D \qquad \textit{and} \qquad \mathcal{C}_{P,R,1} = \bar{x}.$$

Proof. See Appendix A.6 □

The symmetry (e.g. $\mathcal{C}_{R,R,1} = \mathcal{C}_{P,P,1}$ and $\mathcal{G}_{R,1} = \mathcal{G}_{P,1}$) arises because we have assumed the two types have equal utility functions and because the bounds on taxes and entitlements imply that both agents face a common minimum consumption level $\bar{x}$. If we had tightened the upper bound on taxes, for example, allowing rich agents a higher minimum level of consumption, her choices would be significantly different from P's in the first period. While this is an interesting case to study, we leave it for future research. The symmetric case eases the exposition of our results.

The left panel of Figure 6 depicts private consumption allocations in the bargaining equilibrium for proposer i (solid line) and the allocation under no budget rules (dashed line) as functions of q, whereas the right panel depicts public good provision in the two cases, together with the Samuelson level g^*. The figure—which was constructed using the same parameters as Figure 5—illustrates that the proposer has incentives to increase consumption relative to $c_{i,i}^D$ when $q < 1$. As long as she faces uncertainty, the proposer will use taxes or entitlements to ensure consumption above $\bar{x}$ when out of power. This comes at the cost of under-providing public goods (see right panel of the picture) relative to her preferred value of g under certainty. The net benefit derived from distorting the allocations from $c_{i,i}^D$ diminishes in q, and as a result $\mathcal{C}_{i,i,1}$ decreases with the probability of retaining proposal power, q.

6.3 The importance of budget rules

When $q < 1$, and proposers is initially unconstrained (e.g. the acceptance constraint is slack in the first period), both proposer types choose entitlement and tax levels such that $\mathcal{G}_{i,1} < g^*$, indicating that the bargaining equilibrium delivers under-provision of public goods relative to the Samuelson level (dotted blue line in Figure 6). Moreover, aggregate consumption—computed as the sum between the proposer's and the opposition's consumption—is inefficiently large. The degree of inefficiency exacerbates with political uncertainty. In addition to these static sources of inefficiency, individual consumption changes over time, as it varies with the identity of the

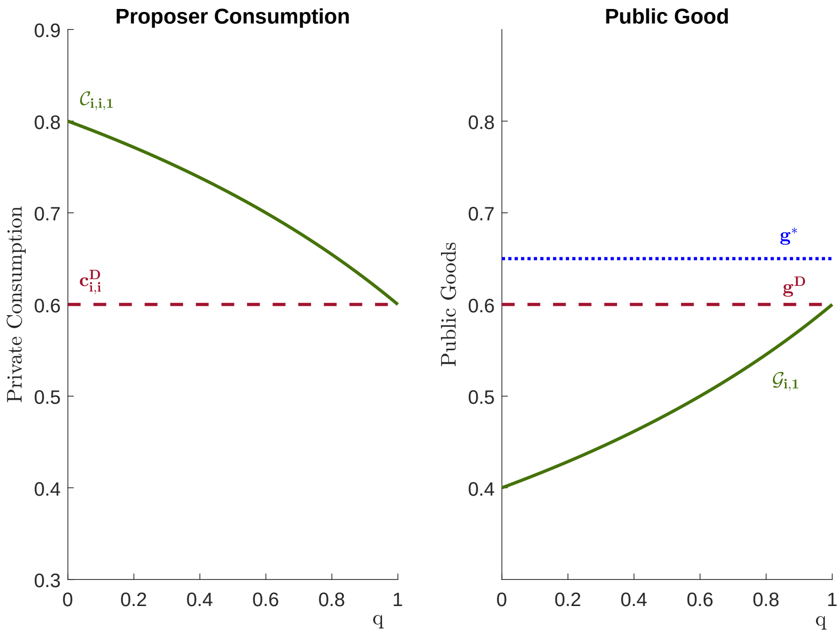


Figure 6: Allocations under budget rules as functions of q.

proposer. Volatility, hence, is a second form of inefficiency of the political equilibrium. Recall that the Planner's solution is time invariant.

What are the implications in terms of equity? Interestingly, when the acceptance constraint is not binding, budget rules exacerbate inequities in the first period.[13] Proposer i sets consumption of the opposition to $\bar{x}$, whereas her own consumption is given by $\mathcal{C}_{i,i,1} > c^D_{i,i}$. As a result, the consumption gap is larger than the one where all spending is discretionary (and taxes are residually determined) for $q < 1$, and increases with higher political turnover. In the second period, the budget rule reduces the consumption gap in expectation. This is because the proposer can actually use her first-mover advantage amid a favorable starting point to lock the society in an equilibrium that favors her. She chooses policy in order to minimize the degree of expropriation of the other party by setting a status quo which ensures consumption tomorrow to exceed $\bar{x}$.

Is the introduction of budget rules, then, beneficial or detrimental to society? It depends, as shown in the following Lemma.

Lemma 1. *Let $q = 0$ and assume P is in power in period 1. Moreover, assume that the acceptance constraint is not binding in $t = 1$. As $x \to 0$, group R is worse off if budget rules are introduced.*

Proof. See Appendix A.7 □

The Lemma highlights that *introducing budget rules does not necessarily lead to Pareto improvements when the acceptance constraint is slack*. In Figure 7 we show, numerically, that this is also the case for $q < 1$ and $\bar{x} > 0$. In particular, we have used $\bar{x} = 0.1$ (and other parameters described in Figure 5). Assuming that P is the proposer in the first period, we plot the lifetime welfare pairs under budget rules (dashed-blue line) vs those under no budget rules (dotted-garnet line). While

[13] We will relax the assumption that the acceptance constraint is slack when we analyze the infinite horizon problem. In the two-period problem, relaxing this constraint is not trivial – and bring little intuition, since there are potentially 8 cases to be analyzed given the bi-dimensionality of the endogenous status quo.

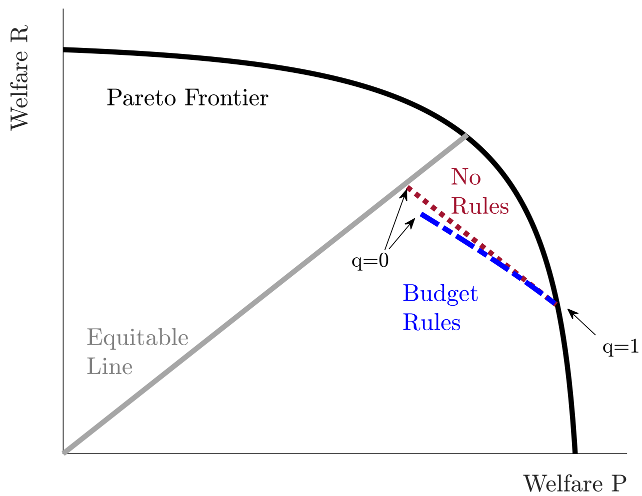


Figure 7: Adopting a budget rule (P in power) for $q \in [0, 1]$

having a budget rule is clearly better for the proposer, the opposition is worse off: the dashed-blue line is at or below the dotted-garnet line. Moreover, the resulting allocations are not only less efficient than the ones under no budget rules (e.g. further away from the Pareto Frontier) but also less equitable (e.g. further away from the Equity line). This is clearest at the point where $q = 0$, where parties alternate in power deterministically. As $q \to 1$, there is less incentive for the first period proposer to take advantage of budget rules to ensure a good bargaining position for the second period, so taxes and entitlements converge to g^D and $c^D_{i,i}$. As a result, the blue and garnet lines approach each other (and get closest to the Pareto frontier). In this example, we see that the introduction of budget rules (a tax code and an entitlement program), from an initial situation without such rules: (i) favors the party that introduces them, (ii) results in less equitable welfare pairs, and (iii) may involve more inefficiencies (e.g. it is further away form the Pareto Frontier).

The status-quo effect: First period allocations were characterized in Proposition 5 under the assumption that proposer P was completely unconstrained (e.g. that the acceptance constraint was slack in $t = 1$). We showed that in such case the proposer would under-provide public goods. This does not necessarily hold when the acceptance constraint binds.

Consider Problem 14, as before, but now subject to the acceptance constraint eq. (9). The latter establishes that the opposition will not accept proposals that make their constituents worse off than under the status-quo policies.

Figure 8 shows the equilibrium level of public good provision in period 1, $\mathcal{G}_{P,1}$ (green line with circles), under an initial status quo value of entitlements that is advantageous for the opposition: $e_0 = 0$. The horizontal axis corresponds to feasible initial status-quo values for the tax code, $\tau_0 = \tau_{sq}$. Note that the proposer no longer has full discretion to choose allocations, as we depart from a scenario without a entitlements. The plot is constructed using the same parameters as Figure 6, but fixing $q = 0.5$. When τ_{sq} is small, public goods are provided at the optimal level g^* (dashed blue line). Party P can no longer get way by taxing R at the maximum possible rate, so in order to be able to increase the level of entitlements, it must offer that the expansion in the size of the government is associated to a rise in g. At the other extreme, when τ_{sq} is higher than a threshold, the proposer can implement the unconstrained solution (solid gray line, $\mathcal{G}^U_{P,1}$)

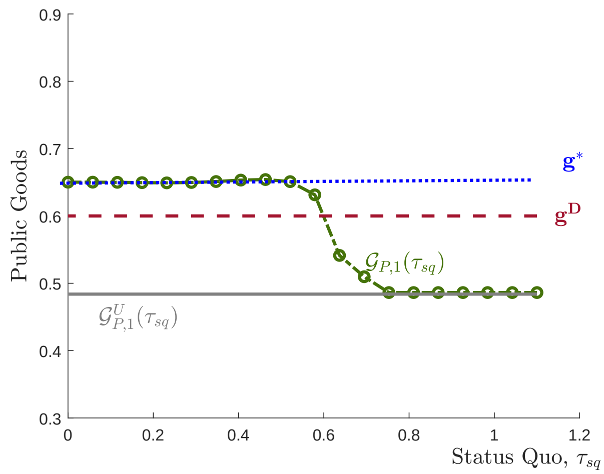


Figure 8: Public good provision as a function of the initial status quo τ_{sq}.

characterized in Proposition 5. The bargaining effect, thus, counteracts the tendency of the budget rule to worsen the under-provision of the public good.

The previous discussion illustrates that whether the introduction of budget rules governing taxes and entitlements lead to Pareto losses or not depends on initial conditions. In the finite horizon model, the proposer starts in an advantageous position when the acceptance constraint is slack because the status quo under the rule will be her most preferred allocation. However, when the acceptance constraint is binding, introducing rules on taxes and entitlements may actually improve welfare, as we will see in the infinite horizon model. Additionally, we could consider alternative decision-making protocols used to introduce these rules. For example, we could analyze an environment where parties bargain on whether to introduce the rules *together* with the initial status-quo allocation under the rule. This scenario is outside the scope of this paper, but it is discussed in detail in Azzimonti et al. (2020).

Entitlements program (no tax code): Would these results be any different if P were to only introduce an entitlement program? Absent a tax code, P would not be subject to the acceptance constraint (13) in the second period. As a result, she would choose g^D, τ_P^D, and e_P^D in $t = 2$. This, in turn, would imply that V_P would be independent of τ_1. Because first period welfare would still be strictly increasing in τ_1, the optimal choice in the first period for a P-proposer would be at the upper bound $\tau_1 = y_R - \bar{x}$. Thus, the existence of a tax code does not change P's incentives to tax R at the maximum feasible rate. The choice of e_1 is also unchanged because in the two period model $W_P(e_1)$ is independent of τ_1. This result is specific to the two period model, as only one of the two state variables is relevant in the second period. In an infinite horizon, however, whether we have both an entitlement program and a tax code, or just one of them would have significant implications for long-run outcomes, as discussed in Section 7.2.

Mandatory spending on g: We could also ask how a budget rule on g, rather than on taxes and entitlements, affects the degree of efficiency and equity of the political equilibrium. This is the case that has been analyzed in detail by Bowen et al. (2014). A key difference between the two budget rules is that while those on τ&e affect private consumption, a budget rule on g affects the provision of public goods. This has important implications for the strategic intertemporal decisions undertaken by poor and rich proposers when in power. We develop this comparison further in Appendix A.8, where we present analytical and quantitative results in an environment closest to Bowen et al. (2014). In particular, we consider utility to be quasi-linear and $\theta \neq 1$.[14] Our main finding is that, under these assumptions, a budget rule on g delivers allocations that put society closer to the optimal solution than budget rules on τ&e do.

That result is, unfortunately, not general. In Section 7.5 we compare the welfare gains (losses) of alternative budget rules in an infinite horizon economy with concave utilities calibrated to the US economy. We find that budget rules on taxes and entitlements bring society closer to optimal than mandatory spending rules on g do. There are two reasons for this apparently contradictory finding. The first one is that consumption profiles are much more volatile under a g-rule. With quasi-linear utility, this factor is irrelevant, as agents do not care about smooth private consumption profiles. These are much more important when utility is concave. The second reason is that the analysis in Appendix A.8 is performed for a two-period model where the incumbent is unconstrained in the first period. Budget rules on taxes and entitlements are associated with significantly more redistribution in the long run, an effect that is abstracted from in a two-period model.

6.4 Unequal Growth

As documented in the introduction, the US experienced a process of "unequal growth" since the 1960s, where most of the gains where enjoyed by the top 50% of income earners and there was little to no growth in the income of individuals in the bottom half of the distribution. In this section, we want to highlight how this affects the level of taxes and entitlements in the bargaining model, and compare them to the optimal allocations.

To capture a process of unequal growth resembling the US experience, we assume that the income of the rich grows unexpectedly between periods 1 and 2: $y_{R,2} > y_{R,1} = y_R$, whereas the income of the poor remains constant: $y_{P,t} = y_P$. As a result, aggregate GDP is higher and inequality rises. The rest of the environment is identical to that in the previous sections. The intuition is cleanest in the case where $\theta = 1$ and utility is logarithmic. Under these assumptions, it is optimal to set public and private consumption according to equation (7). The change in optimal allocations when y_R increases is

$$\frac{\partial g^o}{\partial y_R} = \frac{1}{2} \quad \text{and} \quad \frac{\partial c_i^o}{\partial y_R} = \frac{1}{4}.$$

Half of the the total GDP growth is allocated to public good provision and the rest divided evenly among the two types of agents. Decentralizing this through a tax and entitlement system implies that the size of the government should grow accordingly,

$$\frac{\partial \tau^*}{\partial y_R} = \frac{3}{4} \quad \text{and} \quad \frac{\partial e^*}{\partial y_R} = \frac{1}{4}.$$

Because only the rich become richer in the competitive equilibrium, a planner—who wants to equate the consumption of both agents— will tax the rich at a higher rate so that more public good provision and higher entitlements can be provided.

In the political equilibrium, poor and rich individuals disagree on how to distribute the gains from growth. If a rich proposer were in power in the second period, she would choose a much smaller increase in taxes than optimal, and no expansion of the entitlement program. Inspection

[14] We also briefly discuss the case where utility in both goods is concave in Appendix A.8.2.

of Proposition 4 reveals that when R is unconstrained (e.g. $\bar{e} < e_R^D$)

$$\frac{\partial \Psi_{R,2}(\boldsymbol{s})}{\partial y_R} = \frac{1}{2} \quad \text{and} \quad \frac{\partial \mathcal{E}_{R,2}(\boldsymbol{s})}{\partial y_R} = 0.$$

Hence, a rich agent consumes half of the increase and devotes the rest to additional public good provision. A poor proposer has incentives to tax at a higher rate and to over-provide entitlements. Inspection of Proposition 3 reveals that when P is unconstrained (e.g. $\bar{\tau} > \tau_P^D$), she will choose

$$\frac{\partial \Psi_{P,2}(\boldsymbol{s})}{\partial y_R} = 1 \quad \text{and} \quad \frac{\partial \mathcal{E}_{P,2}(\boldsymbol{s})}{\partial y_R} = \frac{1}{2}.$$

When in power, an unconstrained P proposer would fully expropriate the additional resources and use them to increase her consumption and the provision of public goods.

In this environment, thus, unequal growth paired with a long sequence of periods in which P is in power may explain the growth in the size of the government and the level of entitlements. While this analysis is limited because of the two-period environment, we show that the result holds in an infinite horizon version of our model calibrated to the US experience, developed in Section 7.4.

7 Infinite Horizon Model

The two period model allowed us to build intuition on how budget rules work in a simple environment. We learned that a proposer can strategically use budget rules to position herself advantageously if the opposition were to gain proposal power in the future. In the case of a poor proposer, this is translated into high entitlements. A rich proposer would try to keep taxes and entitlements low, instead. The exact nature of the budget rule, whether the status quo consists of taxes τ, entitlements e, both, or public goods provision alone g, has sharp consequences for the welfare and equity properties of the resulting allocations. Under unequal growth, where only top-income earners see their incomes rise, poor proposers have further incentives to increase the size of the government through expansions of entitlement programs. These results were derived under the extreme assumption that the proposer in period 1 is completely unconstrained (e.g. the acceptance constraint is slack). In this section, we show that an infinite horizon version of the model generates a time path similar to the one in the US data. In particular, as income inequality grows (see Figure 3), the share of entitlements in total public spending also rises (as in Figure 1).

7.1 Parameterization

There is no analytical solution to the proposer's problem in the infinite horizon model. The main reason, and in contrast to most of papers in the bargaining literature, is that we are considering risk-averse agents and multiple endogenous status-quo variables. Our analysis from now on is, thus, numerical. Because of this, we need to choose a reasonable parameterization for the model, which is described next. Our computational strategy is discussed in Appendix A.10.

Utility is logarithmic, $u(x) = \ln(x)$. The value of $\theta = 0.5$ is chosen to match the share of entitlements to total government spending $\frac{e}{e+g} = 0.28$ in 1962. This share is computed using information contained in the historical tables published by the Office of Management and Budget of the White House.[15] The value of g is obtained from Table 8.7, under the item "Total outlays

[15] See `https://www.whitehouse.gov/omb/historical-tables/`

for discretionary programs." The data for e corresponds to the item "Total mandatory programs" in Table 8.5. These are also the series used to construct Figure 1. The discount factor is set to $\beta = 0.97$, consistent with a 3% real interest rate and an annual calibration, implying a standard value for the degree of impatience in the literature. The probability of retaining proposal power is $q = 0.8$, chosen to generate an expected 5-year incumbency by a proposer. This is also the value used in related literature, including Bowen et al. (2014).

Income in our baseline economy is calibrated to match US data during the early 1960s, a low inequality period. Because of this, we will refer to it as the Δ_{low} environment. The value of y_P is chosen to match the pre-tax income (in 100-thousand dollars) of the bottom 50% of earners in the US in 1960, which can be read off Figure 3 (dark red line). The income of the rich is computed such that average income is 0.3, which corresponds to the value (in 100-thousand dollars) attained in 1960 (see green line in Figure 3). This delivers $y_P = 0.1$ and $y_{R,low} = 0.5$, with corresponding inequality of $\Delta_{low} = y_{R,low} - y_L = 0.4$. Minimum consumption is set to $\bar{x} = \bar{x}_g = 0.1$, as in the two period model, in order to ensure a lower bound for entitlement programs equal to zero.[16]

Parameter	Value	Comment
q	0.8	5 year expected incumbency
β	0.97	3% real interest rate
θ	0.5	Share of e in tot spending ($e + g$) in 1960
$\overline{x}$	0.1	Normalize lowest $e_l = 0$
Income		
y_P	0.1	Bottom 50% income share
$y_{R,low}$	0.5	Top 50% income share (1960s), Δ_{low} environment
$y_{R,high}$	1.1	Top 50% income share (2010s), Δ_{high} environment

Table 1: Parameter Values

The main experiment consists on increasing, once and for all, the level of inequality to match the observed value in the US during the early 2010s. We will refer to this as the high inequality, or Δ_{high}, environment. To do so, we consider a process of *unequal growth*, where y_R increases to $y_{R,high} = 1.1$ and y_P stays the same. The income level of the rich is chosen to obtain an average income of 0.6, which corresponds to the value (in 100-thousand dollars) attained in the early 2010 (see blue line in Figure 3). As a result, $\Delta_{high} = 1.1 - 0.1 = 1$ in this scenario.

We assume that the increase in inequality is sudden and unexpected. We do this for two reasons. First, because it greatly simplifies the computation of the bargaining equilibrium: the equilibrium functions for the two regimes (Δ_{low} and Δ_{high}) can be solved for separately.[17] Second, because it allows us to emphasize that budget rules induce an inefficiently slow adjustment in policies. A benevolent planner would jump to the new allocations (and policies which decentralize them) instantaneously.

[16]This choice is not without loss of generality. The larger the value of $\bar{x}$, the less important budget rules will be for welfare. At the other extreme, as $\bar{x} \to 0$, almost any proposal is accepted by the opposition because, due to the logarithmic utility function assumption, not reaching an agreement becomes arbitrarily costly. Given that the optimal solution is constant, it can trivially be implemented with $\bar{x} = g^*$. With this value, policymakers would always be locked at the optimum and the problem would become uninteresting.

[17]If agents were to foresee the increase in inequality, they would adjust policies in advance. The two period model indicates that poor proposers would want to strategically increase entitlements *before* the income of the rich rises. While interesting, this case is significantly more complicated to compute, as there is a change in regime entering expectations.

7.2 Quantitative Analysis

There are two relevant states in the Markov Perfect endogenous bargaining equilibrium, namely last period's taxes $\bar{\tau}$ and entitlements $\bar{e}$. In the two-period model, only one of them was relevant for the proposer: proposer P cared about taxes to the rich whereas proposer R cared about entitlements, because these determined the outside option for the opposition. In the infinite horizon model, on the other hand, both states are relevant when choosing a proposal. To fix ideas, suppose that P is the proposer. The level of taxes established in the code matters because it directly affects how likely the opposition is to accept a change in taxes (as in the two-period example). The value of $\bar{e}$ is also important because, if the proposal is rejected, $\bar{e}$ determines next period's status quo. This, in turn, affects P's bargaining power if R were to become the proposer tomorrow. Through continuation utilities, then, $\bar{e}$ affects today's decisions by proposer P (which did not happen in the finite horizon example).

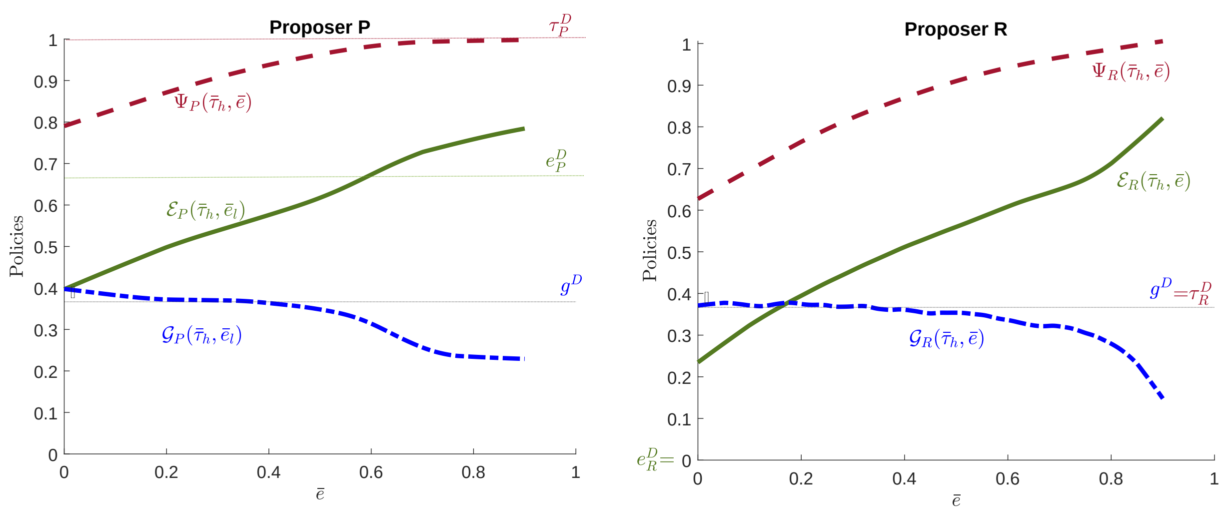


Figure 9: Policy functions for P (left) and R (right) as functions of $\bar{e}$ (fixing $\bar{\tau}_h = 1$). Computed for Δ_{high} environment.

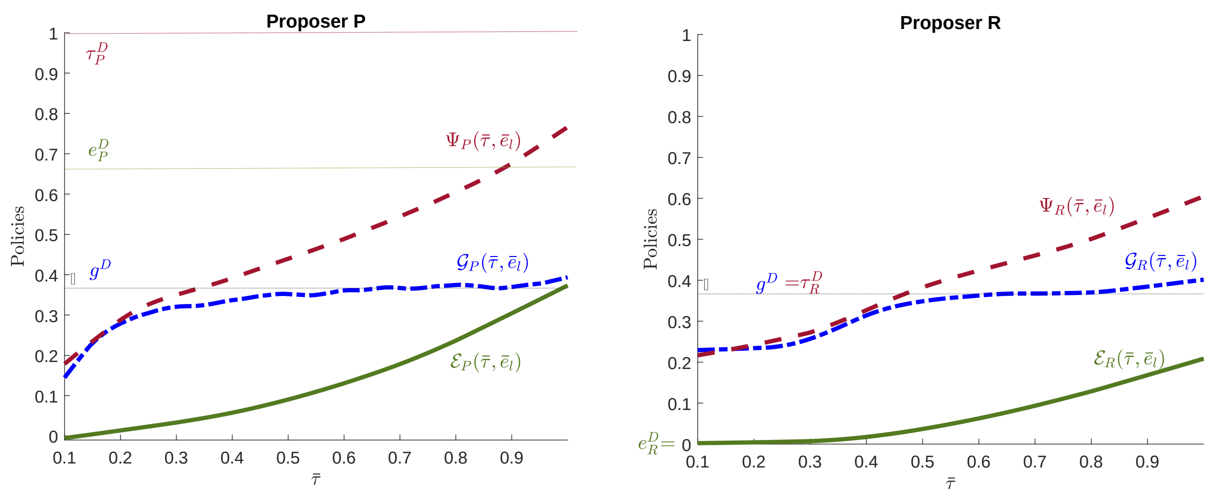
To make the analysis cleaner, we first analyze policy rules as functions of status-quo taxes fixing $\bar{\tau}$, and then let status-quo entitlements vary while fixing taxes. The left panel of Figure 9 depicts P's equilibrium policy rules as functions of $\bar{e}$, fixing taxes at their maximum possible value: $\bar{\tau} = \bar{\tau}_h = 1$.[18]

Taxes (dot-dashed garnet line) and entitlements (dashed green line) are increasing in the status-quo level of entitlements $\bar{e}$, whereas public good provision (solid blue line) declines with it. When $\bar{e} > 0.5$, entitlements are above their value under no rules (e.g, when all spending is discretionary and taxes are residually determined), $\mathcal{E}_P > e_P^D$, whereas public good provision is significantly below it, $\mathcal{G}_P < g^D$. This is in contrast with the two period model, where allocations where independent of $\bar{e}$. That they depend on this state is purely a result of the effect of continuation values on current choices. Incumbent P sets a large entitlement program today in order to make it difficult for the opposition to reduce it in the future. As a result, there is over-provision of private consumption (through entitlements) and under-provision of public goods, even relative to the case with no rules. When $\bar{e}$ is low, P's bargaining power is very limited (remember that entitlements

[18]Throughout, we plot agents' expected choices, prior to the realization of the discrete choice shocks, which we have incorporated for numerical stability purposes. We confirm that, throughout the state space, the choice probabilities' mass is bunched around these expected values, symmetrically and tightly.

are set at zero at the outset of the period), making unconstrained entitlement choices unfeasible. In this situation, R's bargaining power is higher. A poor proposer is forced to offer more public goods and a smaller entitlement program than she would like to, because anything else would be rejected by the opposition.

The right panel shows R's policy variables for the same set of states. Given that status-quo taxes are high (at their maximum values, actually), when entitlements are low, R finds it beneficial to propose a significant tax-cut. P only accepts this in exchange of an expansion of the entitlement program. The expansion is, however, smaller than what would be attained if P were the proposer (to see this, compare the value of entitlements at the origin in the left and right panels). The tax cut is also smaller than R would choose in an unconstrained environment with no rules. This can be seen because τ_R^D is always below $\Psi(\bar{\tau}_h, \bar{e})$. In the infinite horizon model, then, a rich proposer is willing to distort her choices away from those under no rules in order to ensure herself a better barging position in the future. By keeping taxes very low, it can tie the hands of a successor from the P group who wants a larger government size. As the status-quo value of entitlements grows, R finds it more difficult to slash such programs, and finances them at the expense of public goods (that eventually fall below g^D).



(a) Proposer P

(b) Proposer R

Figure 10: Policy functions for P (left) and R (right) as functions of $\bar{\tau}$ (fixing $\bar{e}_l = 0$). Computed for Δ_{high} environment

In Figure 10, we keep status-quo level of entitlements fixed at their minimum feasible level $\bar{e} = \bar{e}_l = 0$ and vary potential status-quo variables for $\bar{\tau}$. Public good provision (solid blue line) increases for low values of $\bar{\tau}$, but remains flat once it reaches g^D. It is also worth pointing out that the unconstrained value of τ_P^D is unfeasible in this scenario. As a result, even when $\bar{\tau}$ is close to the upper bound, a poor proposer is unable to reach an agreement that delivers her preferred value of entitlements, $\mathcal{E}_P < e_P^D$. [19] When $\bar{\tau}$ is low, P's bargaining power is very limited (remember that entitlements are set at zero at the outset of the period). A poor proposer is forced to offer

[19]This result is mainly due to the low value of θ, combined with a high q. We have experimented with scenarios where $\theta = 1$ and $q = 0.5$ in which entitlements can be significantly above their unconstrained value $\mathcal{E}_P > e_P^D$ whereas public good provision can be significantly below it, $\mathcal{G}_P < g^D$.

a minimal entitlement program, because any increase in taxes not used to provide public goods would be rejected by the opposition.

The right hand side of Figure 10 shows the policy rules of a rich proposer that starts the period with $\bar{e} = 0$. Clearly, R has no incentives to start an entitlement program and chooses $\mathcal{E}_R = 0$ for low values of $\bar{\tau}$. Because of the effect of potentially losing elections in the future, she limits the size of the government by choosing lower taxes and public good provision than in the scenario without budget rules. This is an attempt to tie the hands of a potential poor successor who would like to expand the entitlement program. Overall, an R proposer tends to keep the size of the government smaller than a poor one. This can be seen by the fact that all policy rules are flatter under an R proposer.

7.3 Evolution of Policy

The analysis above illustrates that budget rules that make taxes and entitlement programs difficult to change affect a proposers' incentives dynamically. By forcing changes to have a bipartisan support, these rules restrict policymakers' choices, potentially smoothing their evolution over time. Because they have consequences in the future, the strategic incentives may, however, significantly distort the optimal mix between private and public goods. In this subsection, we conduct a series of simulations to further study how policies and allocations evolve over time.

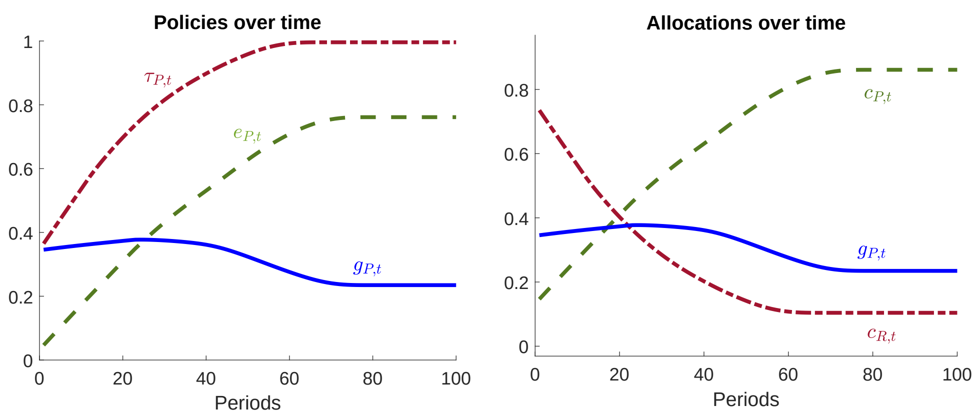


Figure 11: Taxes and entitlements under proposer P. Computed for Δ_{high} environment

When P is in power a long time: We first consider a scenario where proposer P starts period 0 with status-quo taxes $\bar{\tau} = \tau_R^D = 0.366$ and $\bar{e} = e_R^D = 0$, inherited from period -1. This would be the state of the world chosen by R in -1 if unconstrained.[20] We assume that proposer P makes choices under uncertainty, but the realizations of the shock are such that there is no turnover, so P happens to make proposals in all subsequent periods. Policies are shown over time on the left panel of Figure 11, whereas the right panel depicts allocations. Proposer P would like to expand the size of the entitlement program significantly, but it is only able to increase e marginally. The rich are willing to accept this because their consumption of public goods would reduce to $\bar{x}$ if

[20]There are infinitely-many possible initial status-quo values which are part of the ergodic set. To fix ideas, we choose one of them: the one under a discretionary budget.

they were to reject the proposal. To the extent that P remains in power, she will slowly expand the entitlement program—and hence her own consumption of private goods (both dashed green lines),— through rises in taxes. Eventually, P will be able to secure herself a good enough bargaining position (through high $\bar{e}$) to start reducing g and keep high taxes in order to finance increases in e. If P is in power long enough, she would reach a steady-state level of private consumption which is significantly higher than what she would choose under no rules. In that new steady state, public good provision would be inefficient and there would be a high degree of ex-post income inequality, as seen by the distance between the two consumption levels (which reflect after-tax income).

When P and R alternate in power: The economy would only reach a steady state if one of the parties were in power forever. Because the proposer changes stochastically, the direction of policy does as well. Over time, the economy reaches an *ergodic set* determining a range in which policy fluctuates forever-after. To compute the ergodic set, we simulated the economy for $1,000,000$ periods, and eliminated the first $1,000$ periods. It is worth noticing that the economy converges to the same set regardless of initial conditions.

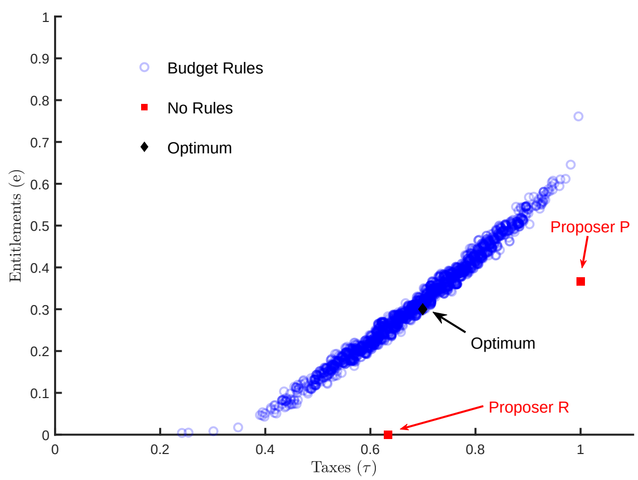


Figure 12: Scatter plot of taxes and entitlements in the simulation. Computed for Δ_{high} environment.

Figure 12 depicts a scatter plot of taxes and entitlement pairs for each period in the simulation (marked with blue circles), together with the pairs that would be obtained under no budget rules for each type of proposer (marked with red squares) and the optimal solution (black diamond). Interestingly, the bargaining solution does not overlap with the solution under no budget rules described in Section 5.2: having to agree on policies reduces the ability of each proposer to reach their most preferred unconstrained value. This brings the solution closer to the optimal one (which is actually reached from time to time in our simulation). On average, taxes, spending, and entitlements (as percentages of total output) are close to the optimal amounts (see last column of Table 2).

Policies span a significant portion of the state-space in the bargaining solution, whereas it would be optimal to keep them constant. This implies that their evolution (and hence, that of private and public consumption) is less persistent and more volatile than optimal, as evident from Table 2. As a result, welfare is slightly below optimal in the decentralized solution with budget rules on taxes and entitlements. More specifically, it is 2.9% lower than welfare attained under optimal allocations.[21] The welfare losses arise for two reasons: (i) slight under-provision of public goods and over-provision of private goods and (ii) volatility in resulting allocations.

	Δ_{low}		Δ_{high}	
	Optimal	Budget Rules on τ&e	Optimal	Budget Rules on τ&e
Long-run Shares (%)				
g/Y	33.3	32.5	33.3	31.1
τ/Y	50.0	49.6	58.3	57.2
e/Y	16.7	17.1	25.0	26.1
$e/(e+g)$	33.3	33.6	42.9	43.9
Coefficient of Variance (%)				
c_P	0	16.8	0	28.4
c_R	0	16.8	0	28.4
g	0	3.4	0	5.2
Autocorrelation (%)				
c_P	100	99.3	100	99.1
c_R	100	99.3	100	99.1
g	100	23.3	100	43.9
Welfare gains				
Per person		-0.24%		-2.9%

Table 2: Moments of the ergodic distribution, budget rules on τ&e vs optimal.

The first two columns of Table 2 report the same long-run statistics but for the baseline economy with Δ_{low}. When inequality is lower, policies are even closer, on average, to the optimal ones. Because volatility is also lower, the welfare attained is just 0.24% below the optimal level. Why does this happen? A potential reason is that there is less room to maneuver in the bargaining solution under Δ_{low}. Given a fixed value of $\bar{x}$, lower inequality implies that agents have a lower ability to exploit budget rules to their advantage. This increases the gridlock region, making changes in policy towards the unconstrained solution less likely.

Finally, Table 2 uncovers a dramatic difference in the long run share of entitlements to total spending $\frac{e}{e+g}$ between the Δ_{low} and the Δ_{high} environments. This suggests that our model could explain the sharp increase in the share of entitlements depicted in Figure 1. We analyze the transition from the low-inequality to the high-inequality environment next.

[21] Welfare gains (and losses) are computed in consumption equivalent terms. Note that $V = (1-\beta)(\log c + \theta \log g) + \beta V$, so average welfare V is equal to $\log c + \theta \log g$. This can be written as $\log(cg^{\theta})$ with cg^{θ} as a "consumption composite." By computing $ce = \exp V$, we obtain welfare levels as consumption equivalent units. In the table, we report the percentage loss between the optimal value ce_o and the one obtained under budget rules ce_r. Welfare gains are computed as $\frac{ce_o - ce_r}{ce_r}$.

7.4 Unequal Growth and Entitlements

The previous section considered two alternative environments, one with low inequality, Δ_{low}, resembling the US in the early 1960s and one with high inequality, Δ_{high}, capturing the US in the 2010s. In this section, we perform the following experiment: suppose that the economy has been in a Δ_{low} environment for a long period of time (so that we are initially somewhere in the ergodic set). Assume that in period 1 there is a permanent and unexpected increase in inequality where only y_R rises (e.g. we move to the Δ_{high} environment). Then, proposers make choices given the new, larger amount of resources. Finally, imagine that proposers alternate in power consistently with the US experience. What would our model predict for the share of entitlements $\frac{e}{e+g}$ over time?

A key calibration detail consists on determining the time-path for alternation of proposers of the two parties during the transition. The US political system has built-in checks and balances: there are two chambers of Congress (the House and the Senate) and a President with veto power. Our model, which follows the bargaining literature, considers a simplified version with just two (unified) proposers, each representing a group. Given this, it is not obvious what the appropriate procedure to match alternation in proposal power in the data would be. In our benchmark, we assume that President's party makes the proposal and the opposition party accepts or rejects. In addition, we consider alternative specifications below, such as a majority in the Senate or in the House, and find that the results are robust.

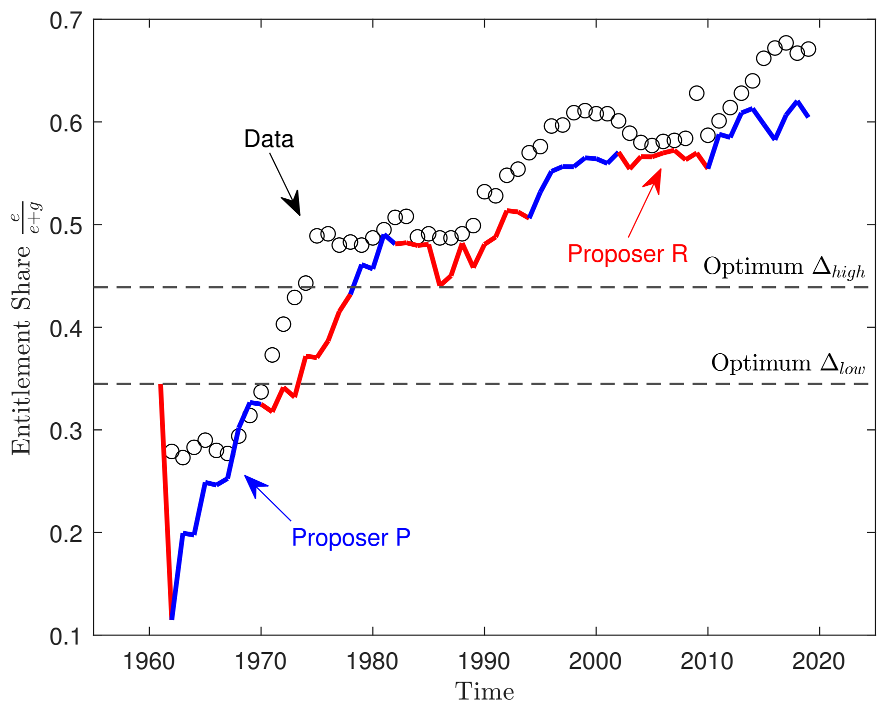
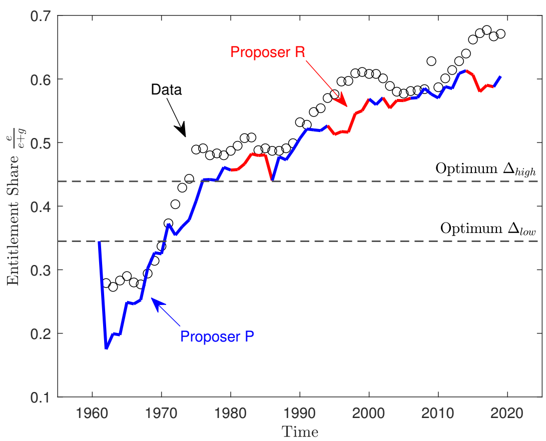


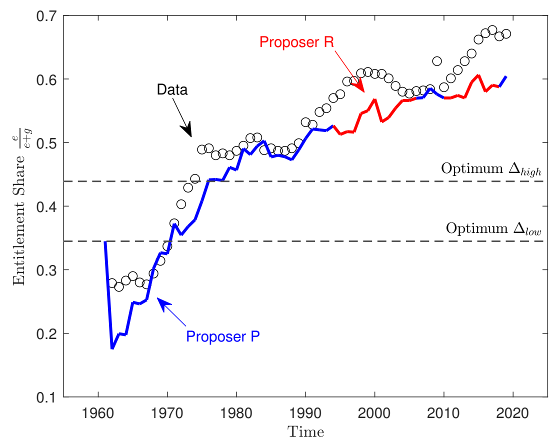
Figure 13: Share of entitlement in total spending under unequal growth.

We assigned proposer power to the President's party between 1960 and 2010, where the Democratic party corresponds to P and the Republican party corresponds to R. This mapping is informed by the perception that Democrats are typically in favor of expanding welfare programs at the expense of discretionary spending (such as defense) and higher taxes, whereas Republicans generally propose tax-cuts and attempt to control the size of the government. Through this procedure, we obtained a sequence of P and R proposers consistent with the data which was inputted in our model simulation. It is worth noting that power fluctuation is uncertain to a proposer making decisions: turnover is always possible with probability q. We simply simulate the economy, given a particular realization of proposal power that mimics the US experience.

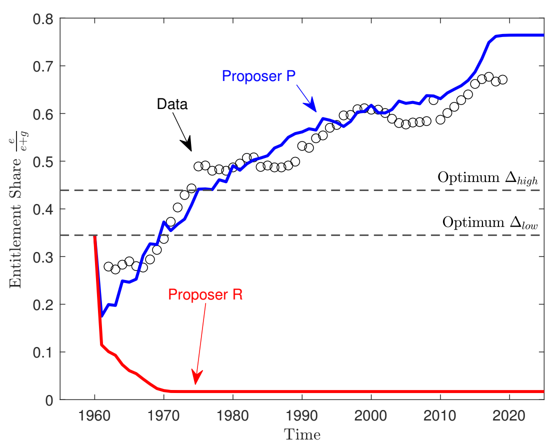
The resulting path for the share of entitlements is shown in Figure 13 (solid line). The corresponding values from the data are represented by circles, mirroring Figure 1. A blue color in the share of entitlements indicates that P has proposer power, whereas a red color indicates that R has proposer power instead. While the fit is imperfect, the model is able to generate an increasing path of the share of entitlements in total spending broadly in line with the data.



(a) Proposer based on majority in the Senate



(b) Proposer based on majority in the House



(c) Unexpected permanent incumbency (R in red, P in blue)

Figure 14: Experiment for alternative proposer sequences

7.4.1 Alternative proposer sequences

For robustness, we run again our model using the time path of proposers based on the majority in: (i) the Senate and (ii) the House of Representatives. The results are shown in the top panels of Figure 14. Except for minor differences, the evolution of the entitlement share is consistent with our previous findings.

The lower panel of Figure 14 shows the evolution of policy assuming, counter-factually, that the same party happens to be the proposer throughout our sample period. During such a lengthy incumbency, proposer R would drive the share of entitlements to zero, despite the increase in inequality.

A benevolent planner would also expand the entitlement program when inequality rises (as shown in the two period model). In our calibrated economy, the planner would jump from the dashed horizontal line in Figure 13 (corresponding to 'Optimum Δ_{low}') to the line labeled 'Optimum Δ_{high}' instantaneously. In the bargaining solution, the transition takes many periods instead. Moreover, we see that the program becomes inefficiently large over time, as the entitlement share is significantly above the optimal level after the early 1980s. In the counterfactual situation in which R is in power throughout the program would instead shrink inefficiently.

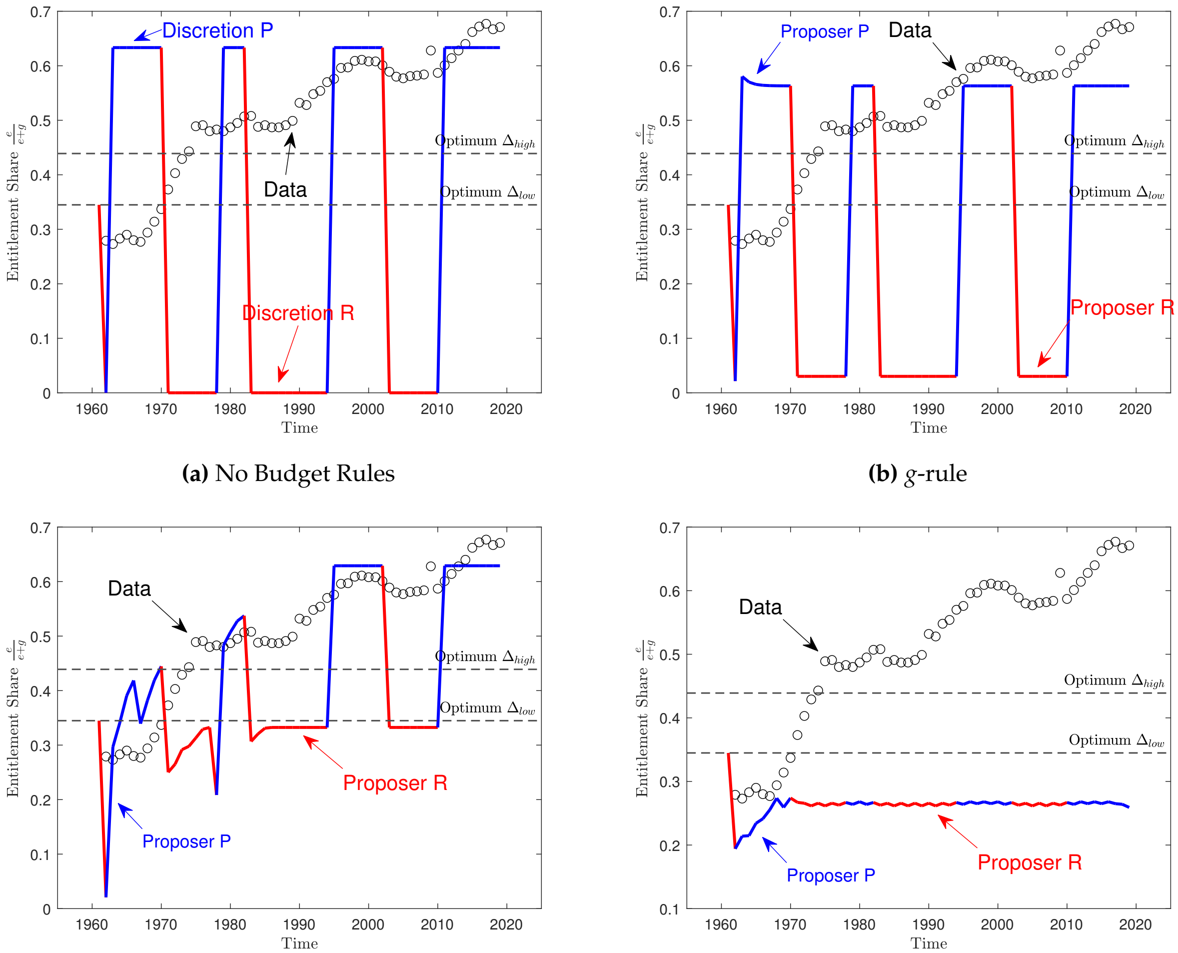
7.4.2 Alternative budgetary arrangements

Would alternative budget rules deliver an evolution of the entitlement share close to the data? We first compute the time path under no budget rules (i.e., where public goods and entitlements are discretionary spending, and taxes are determined residually from the budget constraint). Results are shown in panel (a) of Figure 15. As described in Section 5.2, there are no pre-determined values for policy variables in this case, other than those guaranteeing minimum consumption levels. As a result, any proposal is accepted by the opposition: proposers can always implement their preferred unconstrained levels. Due to the impact of unequal growth, the share of entitlements jump when the poor are in power (as the size of the pie increases and inequality widens), and stays at a constant level until the rich gain control of the government. At that point, the share of entitlement drops to zero. Clearly, this behavior is inconsistent with the evolution of entitlements in the data. Budget rules, by making taxes and entitlements status-quo variables, result in a smoother increase in the early period. Moreover, they do not result in a complete elimination of entitlements every time there is turnover towards party R.[22]

In panel (b) of Figure 15, we plot the case of a budgetary arrangement in which public goods are mandatory spending (e.g., an endogenous status quo variable). The g-rule was introduced in the two period model of Section 6.3 and Appendix A.8. The extension to an infinite horizon model is presented in Appendix A.9. While entitlements are slightly positive when R takes power and slightly lower than the no-rules arrangement when P takes power, the volatility in the entitlement share is significant. Clearly, this rule implies a counter-factual evolution in the period under consideration.

The bottom two panels of the figure display rules where parties bargain over taxes (c) or over entitlements (d) only. Appendix A.9 contains the associated optimization problems for a proposer. Under a τ-rule, transfers to the poor are considered discretionary spending but taxes are not. This presumes the existence of a tax code but no entitlement programs. In contrast to the previous

[22] We think that this experiment illustrates well the importance of bargaining to study fiscal policy in this environment. An environment with voting, either where policies are chosen by the median voter as in Meltzer and Richard (1981) or through probabilistic voting as in Lindbeck and Weibull (1987), would not feature the dynamics perceived in the data.



(a) No Budget Rules

(b) g-rule

(c) τ-rule

(d) e-rule

Figure 15: Alternative budgetary arrangements (proposer based on President's party).

two figures, panel (c) depicts an increasing entitlement share. This happens because the poor are able to keep the government large enough to fund both public goods and private transfers. The volatility, however, is still quite large: entitlements are slashed every time party R gains proposal power. Why are they not eliminated? Because when taxes are initially large, the poor can reject unfavorable proposals and even though this would result in $g = \bar{x}$ and $e = 0$, the rich would still have to pay high taxes, which would be thrown to the ocean. Money-burning gives the poor bargaining power in this scenario, which can be used to ensure positive entitlement spending every period. Clearly, this rule is better for party P than those analyzed in panels (a) and (b). The evolution of entitlements, however, is too volatile in comparison with the US experience.

Finally, in panel (c), we study the case in which entitlements are mandatory but taxes are not. Surprisingly, under an e-rule party P is unable to push the entitlement share up, and the entitlement share remains below its initial value (i.e., pre-growth) throughout the simulation. Why does this happen? Proposer P can pass a policy proposal either because: (i) the threat of public spending collapsing to $\overline{x_g}$ is really bad for R—when the acceptance constraint is slack—or (ii) by offering temporarily higher public spending levels in exchange for an increase in taxes—when the constraint binds. The latter can be used only when g is lower than the value of public goods preferred by the rich under discretion. If the offered g is above such value, the additional increases in taxes needed to fund higher public spending, and possibly additional entitlements, can no longer relax the participation constraint and the proposal fails to pass. Hence, under an e-rule there is an upper bound on how high the status quo value of entitlements can ever reach, because the rich initially have control of resources. By rejecting the proposal, they can simply consume their endowment. The poor need to be able to keep both the level of entitlements and taxes as status quo variables in order to benefit from the bargaining rules. Recall that in the two-period model example of Section 6.4, the poor could implement their desired policies only when $\bar{\tau}$ was large enough; that is, when there were enough resources committed to government spending in the tax code. Absent a tax code, the poor lose an important tool to control how resources are spent.

These results underscore that our benchmark model fits the data remarkably well. We are able to explain the sharp increase in entitlements as a percentage of total spending in the data using a model in which parties bargain over taxes and entitlements. From the figures, we can see that it is the combination of budget rules and persistence of party P with proposal power that leads to such increase.

7.5 The importance of budget rules, revisited

The previous section underscored that alternative budgetary arrangements affect the evolution of policy and the degree of redistribution attained in society after an episode of unequal growth. In this section, we study how alternative budget rules affect welfare in the long run through counterfactual experiments. We consider scenarios in which: (i) there are no budget rules, (ii) taxes are mandatory but entitlements are not (τ-rule), (iii) entitlements are mandatory but taxes are not (e-rule) and (iv) public good spending is mandatory but taxes are not (a g rule). When taxes are not mandatory (e.g there is no tax code), they are set to satisfy the government budget constraint. The maximization problem faced by a proposer, as well as the alternative default values when negotiations break, are summarized in Appendix A.9.

Moments associated with these possibilities are shown in Table 3.[23] Welfare gains (losses, if negative) are computed relative to the optimal value. A political environment without rules is associated with a welfare loss of 32% per-capita, symmetrically distributed across rich and poor agents. As in the two-period model, this arises in part because of the distortions in allocations. Note that g is slightly under-provided and c is slightly over-provided. Most of the losses, however, come from the high volatility of consumption. The optimal solution dictates that c_R and c_P must be constant, but they fluctuate between c_R^D and c_P^D in the scenario with no rules. In this environment, private consumption is valued more than public consumption, so deviations from a smooth profile of c are particularly painful for agents with concave utility. Budget rules on τ&e, our benchmark case, reduce volatility and have more efficient allocations on average (as seen in the third column in the table), reaching welfare levels which are only 3% shy of the optimal ones. For this calibration, then, rules on taxes and entitlements generate a sizable Pareto improvement over the case without rules.

A τ-rule (in the fourth column) would result in welfare gains relative to the no-rules scenario, but it would be dominated by the benchmark case with budget rules on both taxes and entitlements. Interestingly, welfare gains are no longer symmetric. Absent the ability to set an advantageous status-quo for entitlements, poor agents have very limited ability to move the equilibrium in their favor. Rich agents, who start with high wealth, are able to keep most of it by keeping the

[23] All parameters, grids, and algorithm are kept identical across all rules, with one exception: we found that the e-only budget rule requires an increased number of grid points for smooth numerical results. Appendix A.10 documents our computation strategy.

	Optimal	Budget Rules No Rules	τ & e	τ	e	g
Long-run Shares (%)						
g/Y	33.3	30.6	31.1	26.2	33.6	36.2
τ/Y	58.3	56.9	57.2	38.3	45.1	59.8
e/Y	25.0	26.4	26.1	12.1	11.4	23.6
$e/(e+g)$	42.9	31.6	43.9	22.3	24.3	29.6
Coefficient of Variance (%)						
c_P	0	75.9	28.4	68.9	27.0	70.1
c_R	0	75.9	28.4	34.2	12.8	70.1
g	0	0	5.2	19.4	8.1	4.8
Autocorrelation (%)						
c_P	100	60	99	90	98	60
c_R	100	60	99	93	82	60
g	100	100	44	89	64	99
Welfare gains						
Average		−32%	−3%	−20%	−7%	−23%
Rich		−32%	−3%	20%	26%	−23%
Poor		−32%	−3%	−59%	−40%	−23%

Table 3: Alternative budget rules. Computed for Δ_{high} environment

size of the government small. This results in long-run welfare gains of 20% for the rich and losses of almost 60% for the poor. The equilibrium with only a tax code results in a much more unequal distribution of resources. This is in contrast to the two-period analysis, where a τ-only rule was inconsequential for a proposer of type P. It is also different from the simulation in the previous section, where P held proposal power for a long time and attained a large share of entitlements. The big difference arises because welfare involves the entire ergodic set of the equilibrium, underscoring the importance of considering longer horizons in models with endogenous status-quo variables.

An economy with an e-rule (the fifth column) would be better for poor agents than one with only a tax code, but it would still leave them worse off than under our benchmark case. Their welfare is now 40% lower than optimal. Rich agents would again be better off than in the optimal case because this rule reduces the amount of redistribution that would be feasible in the political equilibrium. This result seems, at first, counter-intuitive. Why wouldn't an entitlement rule significantly improve the welfare of agents targeted by such rule? As discussed in the previous section, when R holds proposal power for several periods, the status quo ends up quite unfavorable for poor agents. This is because R manages to cut down entitlements through threats of a "shutdown" on g. When eventually there is turnover and P regains proposal power, the required increases in public spending quickly hit the level beyond which they are ineffective in shifting allocations, before entitlements can be elevated enough to secure a favorable position for P going forward. Why do the poor fare better under this rule than under a τ-rule? After all, we saw in Figure 15 that the share of entitlements was higher than under an e-rule. This happens because poor agents value private and public goods (a variable which was not discussed in the previous section), and $g/Y = 33.6\%$ under the e-rule, versus 26.2% under the τ-rule. Moreover, the volatility of c_P is much smaller under the entitlement-only rule (27% vs 69%).

Finally, we consider a scenario where taxes and entitlements are freely chosen, but public goods are subject to a g-rule. Because both agents derive utility from g, we would expect this rule to be beneficial for society. This is, after all, the main finding from Appendix A.8. Interestingly, we find that it does not achieve a solution closer to the optimal value than the benchmark case, with budget rules in taxes and entitlements. The reason being that public goods are over-provided, while private goods are under-provided in this case. Because agents derive more utility from the consumption of c than from the consumption of g (recall that $\theta = 0.5$ in the calibrated example), this distortion is costly. In addition, while the g rule makes public good provision smoother, with an autocorrelation coefficient of 99%, it makes private consumption more volatile, with an autocorrelation coefficient of 60%, closer to the no-rule scenario. This is a second source of welfare loss in this model. Overall, we find that budget rules over taxes and entitlements allow us to reach a situation which is closer to the optimal one than the other possible rules.[24]

8 Conclusion

We showed that the nature of budget rules, the fiscal instruments constituting the status quo, has first-order consequences for welfare, equity, and thus for the design of optimal budgetary institutions. We leveraged these insights to argue that the increase in the share of entitlements in the US over the last six decades can be rationalized through a political economy model with bargaining over taxes and entitlements, combined with a process of unequal growth.

Using our model, we evaluated how agents fare under counterfactual budget rules. If only public goods were mandatory, welfare of the two types of agents would be lower. This is the case because of the high volatility of allocations that would result in such environment. As expected, eliminating entitlement programs would only benefit high-income earners. Surprisingly, keeping entitlement programs but making taxes easier to change (i.e., no longer a status-quo variable) would have similar effects. This is the case because the rich are initially endowed with more resources than the poor, and would oppose financing redistribution through tax increases over long periods of time. With flexibility in their ability to implement tax-cuts, high-income earners can keep the size of the government small. This shows that generating the dynamics we observe in the data requires budget rules on both taxes and entitlements.

Our model can be extended in several interesting dimensions. We assumed away the distortionary costs of taxation associated to implementing equilibrium allocations. If taxes were distortionary, the marginal gains from expanding entitlement programs could be significantly smaller, plausibly impacting the welfare gains of alternative budget rules. We have also assumed that the government is subject to a balanced budget. The only way in which low-income earners can increase the share of entitlements or high-income earners introduce a tax-cut is by reducing the provision of public goods. If the government were able to issue debt, we would expect parties to potentially agree to borrow in order to achieve their desired allocations. However, as Bouton et al. (2020) pointed out, entitlement programs may be a good substitute for debt in these types of models. Thus, it would be interesting to augment our model to consider the possibility of issuing public debt in order to study whether and how policymakers would use all policy instruments in the political equilibrium.

[24] This last result hinges on the specific parameterization. In an environment where θ is higher and q is lower, a mandatory spending rule on g could plausibly generate higher welfare gains (details available upon request).

We have considered a situation where the income level of each agent is fully persistent over time. This makes the preferred fiscal policy of each group starkly different. However, if agents were subject to idiosyncratic risk, entitlement programs would also provide some degree of insurance. Other possible extensions involve studying how volatility in the endowment or changes in the degree of income inequality could affect our findings.[25]

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[25]See Dziuda and Loeper (2016) and Dziuda and Loeper (2018) for environments where policymakers are subject to preference shocks.

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A Appendix

A.1 Proof of Proposition 1.

The set of efficient allocations $\mathbf{a}^*$ can be traced-out by choosing sequences of private and public consumption in order to maximize a weighted sum of the lifetime utility of agents,

$$\max_{\{\mathbf{a}\}} \left\{ \lambda \mathcal{V}_P(\mathbf{a}) + (1-\lambda)\mathcal{V}_R(\mathbf{a}) \right\}, \tag{16}$$

subject to the resource constraint, eq. (5). The parameter $\lambda \in (0,1)$ denotes the Pareto-weight of poor agents. For $\lambda \in (0,1)$, the Lagrangian is given by:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \lambda U(c_{P,t}, g_t) + (1-\lambda)U(c_{R,t}, g_t) + \psi_t \left[Y - c_{P,t} - c_{R,t} - g_t \right] \right\}$$

Recalling $U(c_{i,t}, g_t) = u(c_{i,t}) + \theta u(g_t)$, the first order and Kuhn-Tucker conditions for this problem are $c_{P,t}, c_{R,t}, g_t, \psi_t \geq 0$ and

$$\begin{aligned}
[c_{P,t}] \quad & \lambda u'(c_{P,t}) - \psi_t \leq 0 \\
& c_{P,t}[\lambda u'(c_{P,t}) - \psi_t] = 0 && (17) \\
[c_{R,t}] \quad & (1-\lambda)u'(c_{R,t}) - \psi_t \leq 0 \\
& c_{R,t}[(1-\lambda)u'(c_{R,t}) - \psi_t] = 0 && (18) \\
[g_t] \quad & \theta u'(g_t) - \psi_t \leq 0 \\
& g_t[\theta u'(g_t) - \psi_t] = 0 && (19) \\
[RC] \quad & [Y - c_{P,t} - c_{R,t} - g_t]\psi_t = 0 && (20)
\end{aligned}$$

Since $u(.)$ is increasing in its argument, $\psi_t > 0$ and eq. (20) holds with equality. Because $u(.)$ satisfies Inada conditions and $\lambda \in (0,1)$, $g_t, c_{P,t}, c_{R,t} > 0$. This implies that

$$(1-\lambda)u'(c^*_{R,t}) = \lambda u'(c^*_{P,t}) = \theta u'(g^*_t).$$

Under logarithmic utility,

$$\frac{\theta}{g^*_t} = \frac{\lambda}{c^*_{P,t}} = \frac{1-\lambda}{c^*_{R,t}}.$$

which, after some manipulation, delivers the expressions in the main text.

A.2 Proof to Proposition 2

Because taxes and transfers are non-distortionary, the maximization problem is equivalent to one where the government chooses allocations directly. In other words, we can re-write the problem as

$$V^D = \max_{\{c_{i,i}, c_{j,i}, g\}} \left\{ u(c_{i,i}) + u(g) + \beta \left[qV^D + (1-q)W^D \right] \right\}, \tag{21}$$

subject to the resource constraint (5) and the bounds on consumption $c_{i,i}, c_{i,j} \geq \bar{x}$. Moreover, the problem of an incumbent is independent of her type when written in terms of allocations, so $V^D_i = V^D$. Because there is no dynamic state variable, the problem is equivalent to

$$\max_{\{c_{j,i}, g\}} \left\{ u(Y - c_{j,i} - g) + u(g) \right\}$$

subject to $c_{j,i} \geq \bar{x}$. Clearly, it is optimal for an incumbent to set $c_{j,i} = \bar{x}$. The optimal g equates her private marginal benefit of g to her marginal cost. Taking first order conditions yields the result in the proposition. Policies can be decentralized using eq. (4).

A.3 Proof of Proposition 3

Let $u(x) = \ln(x)$. Since endowments and taxes are non-distortionary, we can solve the problem in terms of allocations. This requires a transformation for the state space from status-quo policy pairs $\{\overline{\tau}, \overline{e}\}$ into status-quo consumption levels $\{\overline{c_P}, \overline{c_R}\}$. The consumer budget constraints provide a simple linear mapping to do this. Given $\mathbf{s} = \{\overline{c_P}, \overline{c_R}\}$, party's P Lagrangian for this problem at $t = 2$ is given by:

$$\mathcal{L} = \ln(c_{P,2}) + \ln(g_2) + \lambda\left[Y - c_{P,2} - c_{R,2} - g_2\right] + \psi\left(\ln(c_{R,2}) + \ln(g_2) - \ln(\overline{c_R}) - \ln(\overline{x})\right)$$

The first-order and Kuhn-Tucker conditions are $c_{P,2}, c_{R,2}, g_2 \geq \overline{x}$, $\lambda, \psi \geq 0$ and

$$\begin{aligned}
&[c_{P,2}] \quad \frac{1}{c_{P,2}} - \lambda \leq 0. \\
&\qquad (c_{P,2} - \bar{x})\left[\frac{1}{c_{P,2}} - \lambda\right] = 0. && (22) \\
&[c_{R,2}] \quad \frac{\psi}{c_{R,2}} - \lambda \leq 0. \\
&\qquad (c_{R,2} - \bar{x})\left[\frac{\psi}{c_{R,2}} - \lambda\right] = 0. && (23) \\
&[g_2] \quad \frac{1+\psi}{g_2} - \lambda \leq 0. \\
&\qquad (g_2 - \bar{x})\left[\frac{1+\psi}{g_2} - \lambda\right] = 0. && (24) \\
&[RC] \quad [Y - c_{P,2} - c_{R,2} - g_2] \geq 0. \\
&\qquad \lambda\left[Y - c_{P,2} - c_{R,2} - g_2\right] = 0. && (25) \\
&[IRC] \quad [\ln(c_{R,2}) + \ln(g_2) - \ln(\overline{c_R}) - \ln(\overline{x})] \geq 0. \\
&\qquad \psi\left[\ln(c_{R,2}) + \ln(g_2) - \ln(\overline{c_R}) - \ln(\overline{x})\right] = 0. && (26)
\end{aligned}$$

The solution to this system depends on the status quo vector $\mathbf{s} = \{\overline{c_R}, \overline{c_P}\}$. Denote the solution by functions $\mathcal{C}_{i,P,2}(\mathbf{s})$ for private consumption, $\mathcal{G}_{P,2}(\mathbf{s})$ for public goods, and $\Psi_{P,2}(\mathbf{s})$ and $\mathcal{E}_{P,2}(\mathbf{s})$ for policy variables.

First note that $\lambda > 0$, since $u(.)$ is increasing in its arguments. In addition, since we assume $\overline{x}$ is relatively small relative to Y, $\mathcal{C}_{i,P,2}(\mathbf{s}), \mathcal{G}_{P,2}(\mathbf{s}) > \overline{x}$. We have three cases to consider:

- Case 1: $\psi = 0$. By eq. (23) we have that $\mathcal{C}_{R,P,2}(\mathbf{s}) = \overline{x}$. By eq. (22), eq. (24) and eq. (25), we have that $\mathcal{C}_{P,P,2}(\mathbf{s}) = \mathcal{G}_{P,2}(\mathbf{s}) = \frac{Y-\overline{x}}{2}$. Using eq. (3) to solve for transfers delivers $\mathcal{E}_{P,2}(\mathbf{s}) = \frac{\Delta-\overline{x}}{2} = e_P^D$. The expression for $\Psi_{P,2}(\mathbf{s})$ can be obtained from eq. (2). By eq. (26), this case holds if and only if $\overline{c_R} = y_R - \overline{\tau} < \frac{Y-\overline{x}}{2}$. In terms of policies, this happens when $\overline{\tau} > \frac{\Delta+\overline{x}}{2} = \tau_R^D$.
- Case 2: $\psi > 0$, $\mathcal{C}_{R,P,2}(\mathbf{s}) = \overline{x}$ and $\mathcal{C}_{P,P,2}(\mathbf{s}), \mathcal{G}_{P,2}(\mathbf{s}) > \overline{x}$. By eq. (26) we have that $\mathcal{G}_{P,2}(\mathbf{s}) = \overline{c_R} = y_R - \overline{\tau}$. By eq. (25), we have that $\mathcal{C}_{P,P,2}(\mathbf{s}) = Y - \overline{x} - \overline{c_R}$. From eq. (3), we have that $\mathcal{E}_{P,2}(\mathbf{s}) = \overline{\tau} - \overline{x}$. From eq. (2), $\Psi_{P,2}(\mathbf{s}) = \tau_P^D$. By eq. (22), eq. (24), eq. (23) and the fact that $\mathcal{C}_{R,P,2}(\mathbf{s}) \geq \overline{x}$, this case holds if and only if $\frac{\Delta}{2} < \overline{\tau} \leq \tau_R^D$.

- Case 3: $\psi > 0$ and $\mathcal{C}_{i,P,2}(\mathbf{s}), \mathcal{G}_{P,2}(\mathbf{s}) > \overline{x}$. By eq. (22), eq. (23), eq. (24) and eq. (25), we have that $\mathcal{G}_{P,2}(\mathbf{s}) = \frac{Y}{2} = g^*$. By eq. (26), we have that $\mathcal{C}_{R,P,2}(\mathbf{s}) = \frac{2\overline{x}\overline{c_R}}{Y}$. From eq. (2), we have that $\Psi_{P,2}(\mathbf{s}) = y_R - \frac{2\overline{x}\overline{c_R}}{Y} = y_R - \frac{2\overline{x}(y_R - \overline{\tau})}{Y}$. The expression for $\mathcal{E}_{P,2}(\mathbf{s})$ follows by using eq. (3). By eq. (22), eq. (24) and eq. (23), this case holds if and only if $\overline{\tau} < \frac{\Delta}{2}$.

A.4 Proof of Proposition 4

Party's R Lagrangian for this problem at $t = 2$ is given by:

$$\mathcal{L} = \ln(c_{R,2}) + \ln(g_2) + \lambda \left[Y - c_{P,2} - c_{R,2} - g_2\right] + \psi \left(\ln(c_{P,2}) + \ln(g_2) - \ln(\overline{c_P}) - \ln(\overline{x})\right)$$

The first-order and Kuhn-Tucker conditions party R are $c_{P,2}, c_{R,2}, g_2 \geq \overline{x}$, $\lambda, \psi \geq 0$ and

$$[c_{P,2}] \quad \frac{\psi}{c_{P,2}} - \lambda \leq 0.$$

$$(c_{P,2} - \bar{x}) \left[\frac{\psi}{c_{P,2}} - \lambda\right] = 0. \tag{27}$$

$$[c_{R,2}] \quad \frac{1}{c_{R,2}} - \lambda \leq 0.$$

$$(c_{R,2} - \bar{x}) \left[\frac{1}{c_{R,2}} - \lambda\right] = 0. \tag{28}$$

$$[g_2] \quad \frac{1+\psi}{g_2} - \lambda \leq 0.$$

$$(g_2 - \bar{x}) \left[\frac{1+\psi}{g_2} - \lambda\right] = 0. \tag{29}$$

$$[RC] \quad \left[Y - c_{P,2} - c_{R,2} - g_2\right] \geq 0.$$

$$\lambda \left[Y - c_{P,2} - c_{R,2} - g_2\right] = 0. \tag{30}$$

$$[IRC] \quad \left[\ln(c_{P,2}) + \ln(g_2) - \ln(\overline{c_P}) - \ln(\overline{x})\right] \geq 0.$$

$$\psi \left[\ln(c_{P,2}) + \ln(g_2) - \ln(\overline{c_P}) - \ln(\overline{x})\right] = 0. \tag{31}$$

First note that $\lambda > 0$, since $u(.)$ is increasing in its arguments. Also, since we assume $\overline{x}$ is relatively small relative to Y, $\mathcal{C}_{i,R,2}(\mathbf{s}), \mathcal{G}_{R,2}(\mathbf{s}) > \overline{x}$. We have three cases to consider:

- Case 1: $\psi = 0$. By eq. (27) we have that $\mathcal{C}_{P,R,2}(\mathbf{s}) = \overline{x}$. By eq. (28), eq. (29) and eq. (30), we have that $\mathcal{C}_{R,R,2}(\mathbf{s}) = \mathcal{G}_{R,2}(\mathbf{s}) = \frac{Y - \overline{x}}{2}$. The expressions for $\mathcal{E}_{R,2}(\mathbf{s})$ and $\Psi_{R,2}(\mathbf{s})$ can be obtained by replacing allocations into the consumers' budget constraints. By eq. (31), this case holds if and only if $\overline{c_P} = y_P + \overline{e} < \frac{Y - \overline{x}}{2}$. This happens only when $\overline{e} < \frac{\Delta - \overline{x}}{2} = e_P^D$.

- Case 2: $\psi > 0$, $\mathcal{C}_{P,R,2}(\mathbf{s}) = \overline{x}$ and $\mathcal{C}_{R,R,2}(\mathbf{s}), \mathcal{G}_{R,2}(\mathbf{s}) > \overline{x}$. By eq. (26) we have that $\mathcal{G}_{R,2}(\mathbf{s}) = \overline{c_P} = y_P + \overline{e}$. By eq. (25), we have that $\mathcal{C}_{R,R,2}(\mathbf{s}) = Y - \overline{x} - \overline{c_P}$. The expressions for $\mathcal{E}_{R,2}(\mathbf{s})$ and $\Psi_{R,2}(\mathbf{s})$ can be obtained by replacing allocations into the consumers' budget constraints. By eq. (22), eq. (24), eq. (23) and the fact that $\mathcal{C}_{P,R,2}(\mathbf{s}) \geq \overline{x}$, this case holds if and only if $\frac{\Delta - \overline{x}}{2} = e_P^D < \overline{e} \leq \frac{\Delta}{2}$.

- Case 3: $\psi > 0$ and $\mathcal{C}_{i,R,2}(\mathbf{s}), \mathcal{G}_{R,2}(\mathbf{s}) > \overline{x}$. By eq. (22), eq. (23), eq. (24) and eq. (25), we have that $\mathcal{G}_{R,2}(\mathbf{s}) = \frac{Y}{2} = g^*$. By eq. (26), we have that $\mathcal{C}_{P,R,2}(\mathbf{s}) = \frac{2\overline{x}\overline{c_P}}{Y}$. The expressions for $\mathcal{E}_{R,2}(\mathbf{s})$ and $\Psi_{R,2}(\mathbf{s})$ can be obtained by replacing allocations into the consumers' budget constraints. By eq. (22), eq. (24) and eq. (23), this case holds if and only if $\overline{e} \geq \frac{\Delta}{2}$.

A.5 Proof of Proposition 5

Since taxes and entitlements are all non-distortionary, we can write the whole problem in terms of allocations. Continuation values in terms of allocations satisfy:

$$V_P(c_{R,1}) = \begin{cases} \ln\left(\frac{Y-\bar{x}}{2}\right) + \ln\left(\frac{Y-\bar{x}}{2}\right), & \text{if} \quad c_{R,1} < \frac{Y-\bar{x}}{2}. \\ \ln(Y-\bar{x}-c_{R,1}) + \ln(c_{R,1}), & \text{if} \quad c_{R,1} \in \left[\frac{Y-\bar{x}}{2}, \frac{Y}{2}\right) \\ \ln\left(\frac{Y}{2} - \frac{2\bar{x}c_{R,1}}{Y}\right) + \ln\left(\frac{Y}{2}\right), & \text{if} \quad c_{R,1} \geq \frac{Y}{2} \end{cases}$$

and

$$W_P(c_{P,1}) = \begin{cases} \ln(\bar{x}) + \ln\left(\frac{Y-\bar{x}}{2}\right), & \text{if} \quad c_{P,1} < \frac{Y-\bar{x}}{2}. \\ \ln(\bar{x}) + \ln(c_{P,1}), & \text{if} \quad c_{P,1} \in \left[\frac{Y-\bar{x}}{2}, \frac{Y}{2}\right) \\ \ln\left(\frac{2\bar{x}c_{P,1}}{Y}\right) + \ln\left(\frac{Y}{2}\right), & \text{if} \quad c_{P,1} \geq \frac{Y}{2} \end{cases}$$

Note that the only relevant state for party P when in power is $c_{R,1}$ and for when out of power is $c_{P,1}$. Therefore, party's P Lagrangian for this problem at $t = 1$ is given by:

$$\mathcal{L} = \ln(c_{P,1}) + \ln(g_1) + \beta\left[qV_P(c_{R,1}) + (1-q)W_P(c_{P,1})\right] + \lambda\left[Y - c_{P,1} - c_{R,1} - g_1\right]$$

The first-order and Kuhn-Tucker conditions party P are $c_{P,1}, c_{R,1}, g_1 \geq \bar{x}$, $\lambda, \psi \geq 0$ and

$$\begin{aligned} [c_{P,1}] \quad & \frac{1}{c_{P,1}} + \beta(1-q)\frac{dW^P}{dc_{P,1}} - \lambda \leq 0. \\ & (c_{P,1} - \bar{x})\left[\frac{1}{c_{P,1}} + \beta(1-q)\frac{dW^P}{dc_{P,1}} - \lambda\right] = 0. \end{aligned} \tag{32}$$

$$\begin{aligned} [c_{R,1}] \quad & \beta q\frac{dV^P}{dc_{R,1}} - \lambda \leq 0. \\ & (c_{R,1} - \bar{x})\left[\beta q\frac{dV^P}{dc_{R,1}} - \lambda\right] = 0. \end{aligned} \tag{33}$$

$$\begin{aligned} [g_1] \quad & \frac{1}{g_1} - \lambda \leq 0. \\ & (g_1 - \bar{x})\left[\frac{1}{g_1} - \lambda\right] = 0. \end{aligned} \tag{34}$$

$$\begin{aligned} [RC] \quad & [Y - c_{P,1} - c_{R,1} - g_1] \geq 0. \\ & \lambda\left[Y - c_{P,1} - c_{R,1} - g_1\right] = 0. \end{aligned} \tag{35}$$

Again, first note that $\lambda > 0$, since $u(.)$ is increasing in its arguments. Also, since we assume $\bar{x}$ is relatively small relative to Y, $\mathcal{C}_{P,P,1}, \mathcal{G}_{P,1} > \bar{x}$. Also, by eq. (33), we have that $\mathcal{C}_{R,P,1} = \bar{x}$. This implies that $\frac{dV^P(c_{R,1})}{dc_{R,1}} = 0$. By eq. (35), we have that $c_{P,1} = Y - \bar{x} - g_1$ and $\frac{dW^P(c_{P,1})}{dc_{P,1}} = \frac{1}{c_{P,1}}$. By eq. (32) and eq. (34), we have that $\mathcal{G}_{P,1} = \frac{Y-\bar{x}}{2+\beta(1-q)} = \frac{2}{2+\beta(1-q)}g^D$. Back to eq. (35), we have that $\mathcal{C}_{P,P,1} = (1+\beta(1-q))\,\frac{Y-\bar{x}}{2+\beta(1-q)}$. Since $\mathcal{C}_{P,P,1} = y_P + \mathcal{E}_{P,1}$ we have that $\mathcal{E}_{P,1} = \frac{2e_P^D + \beta(1-q)\tau_D^P}{2+\beta(1-q)}$. Also, since $\mathcal{C}_{R,P,1} = \bar{x} = y_R + \Psi_{P,1}$, we have that $\Psi_{P,1} = y_R - \bar{x} = \tau_P^D$.

A.6 Proof of Proposition 6

We follow as for the problem of party P. Party's R Lagrangian for this problem at $t = 1$ is given by:

$$\mathcal{L} = \ln(c_{R,1}) + \ln(g_1) + \beta\left[qV_R(c_{P,1}) + (1-q)W_R(c_{R,1})\right] + \lambda\left[Y - c_{P,1} - c_{R,1} - g_1\right]$$

The first-order and Kuhn-Tucker conditions party R are $c_{P,1}, c_{R,1}, g_1 \geq \overline{x}$, $\lambda, \psi \geq 0$ and

$$\begin{aligned}
[c_{R,1}] \quad & \frac{1}{c_{R,1}} + \beta(1-q)\frac{dW^R}{dc_{R,1}} - \lambda \leq 0. \\
& (c_{R,1} - \bar{x})\left[\frac{1}{c_{R,1}} + \beta(1-q)\frac{dW^R}{dc_{R,1}} - \lambda\right] = 0. && (36)\\
[c_{P,1}] \quad & \beta q\frac{dV^R}{dc_{P,1}} - \lambda \leq 0. \\
& (c_{P,1} - \bar{x})\left[\beta q\frac{dV^R}{dc_{P,1}} - \lambda\right] = 0. && (37)\\
[g_1] \quad & \frac{1}{g_1} - \lambda \leq 0. \\
& (g_1 - \bar{x})\left[\frac{1}{g_1} - \lambda\right] = 0. && (38)\\
[RC] \quad & \left[Y - c_{P,1} - c_{R,1} - g_1\right] \geq 0. \\
& \lambda\left[Y - c_{P,1} - c_{R,1} - g_1\right] = 0. && (39)
\end{aligned}$$

Again, first note that $\lambda > 0$, since $u(.)$ is increasing in its arguments. Also, since we assume $\overline{x}$ is relatively small relative to Y, $\mathcal{C}_{R,R,1}, \mathcal{G}_{R,1} > \overline{x}$. Also, by eq. (37), we have that $\mathcal{C}_{P,R,1} = \overline{x}$. This implies that $\frac{dV^R(c_{P,1})}{dc_{P,1}} = 0$. By eq. (35), we have that $c_{R,1} = Y - \overline{x} - g_1$ and $\frac{dW^R(c_{R,1})}{dc_{R,1}} = \frac{1}{c_{R,1}}$. By eq. (36) and eq. (38), we have that $\mathcal{G}_{R,1} = \frac{Y-\overline{x}}{2+\beta(1-q)} = \frac{2}{2+\beta(1-q)}g^D$, as in party's P problem. Back to eq. (35), we have that $\mathcal{C}_{R,R,1} = (1+\beta(1-q))\frac{Y-\overline{x}}{2+\beta(1-q)}$. Since $\mathcal{C}_{R,R,1} = y_R - \Psi_1$ we have that $\Psi_1 = \frac{y_R+(1+\beta(1-q))e^D_R}{2+\beta(1-q)}$. Also, since $\mathcal{C}_{P,R,1} = \overline{x} = y_P + \mathcal{E}_{R,1}$, we have that $\mathcal{E}_{R,1} = \overline{x} - y_P = e^D_R$.

A.7 Proof to Lemma 1

Let P be the proposer in the first period, since $q = 0$, R is the proposer in $t = 2$. When all spending is discretionary and taxes are residually determined, the lifetime welfare of R and P are given, respectively, by

$$U^D_P = (2+\beta)\ln(c^D_{P,P}) + \beta\ln(\bar{x})$$

$$U^D_R = \ln(\bar{x}) + (1+2\beta)\ln(c^D_{R,R})$$

where $c^D_{i,i} = \frac{Y-\bar{x}}{2}$, as shown in previous results. To find lifetime welfare under budget rules, first use Proposition 5, evaluated at $q = 0$, to find first period allocations.

$$\mathcal{C}_{P,P,1} = \frac{2(1+\beta)}{2+\beta}c^D_{P,P} \quad \mathcal{G}_{P,1} = \frac{2}{2+\beta}c^D_{P,P} \quad \text{and} \quad \mathcal{C}_{R,P,1} = \bar{x}.$$

Under the assumption that $\bar{x} \to 0$, the relevant state variable for proposer R at the outset of the second period is given by $c_{P,1} = \mathcal{C}_{P,P,1} \geq \frac{Y}{2}$. Hence, second period allocations satisfy

$$\mathcal{C}_{P,R,2} = \frac{2\bar{x}c_{P,1}}{Y} \quad \mathcal{G}_{R,2} = \frac{Y}{2} \quad \text{and} \quad \mathcal{C}_{R,R,2} = \frac{Y}{2} - \frac{2\bar{x}c_{P,1}}{Y}.$$

The lifetime welfare of R, under budget rules, is thus

$$U_R^{BR} = \ln(\bar{x}) + \ln\left(\frac{2}{2+\beta}c_{P,P}^D\right) + \beta\left\{\ln\left(\frac{Y}{2}\right) + \ln\left(\frac{Y}{2} - \frac{2\bar{x}c_{P,1}}{Y}\right)\right\}$$

Noting that $\frac{Y}{2} - \frac{2\bar{x}c_{P,1}}{Y} = \frac{Y}{2}\left[1 - \frac{4\bar{x}c_{P,1}}{Y^2}\right]$, and simplifying, the welfare gain obtained by R from imposing budget rules is given by

$$U_R^{BR} - U_R^D = \ln\left(\frac{2}{2+\beta}\right) + 2\beta\left[\ln\left(\frac{Y}{2}\right) - \ln(c_{R,R}^D)\right] + \beta\ln\left(1 - \frac{4\bar{x}c_{P,1}}{Y^2}\right).$$

The values inside the first and last terms are smaller than 1, so their natural logarithm is negative. Taking limits when $\bar{x} \to 0$, the second and third terms go to 0 since $c_{R,R}^D = \frac{Y-\bar{x}}{2}$. As a result, R's lifetime welfare is lower under budget rules than in the case without rules. Numerically, it is easy to see that this holds for $\bar{x} > 0$.

A.8 Mandatory Spending on Private vs Public Goods

Bowen et al. (2014) show that mandatory spending on public goods can prevent under-provision of public goods that happen when all spending is discretionary. Moreover, they derive conditions under which such budget rule can restore efficiency in a bargaining equilibrium. We show that the type of budget rule matters for whether mandatory spending is beneficial or detrimental for society. In this section, we compare two types of rules: (1) mandatory spending on public goods (*g-rule*) versus (2) a tax code and a mandatory spending rule in entitlements (τ&*e-rule*). The first rule is the one studied in detail in Bowen et al. (2014). The second one is our benchmark case. The key difference between them is that (1) targets public consumption whereas (2) targets private consumption. The objective of this section is to compare how these rules affect the political equilibrium.

It is easy to see that the tax code establishing a status-quo value for τ is equivalent to a rule establishing a status-quo value on the level of consumption of the rich. This is the case because $c_R = y_R - \tau$, and y_R is exogenous (the equivalence would break if agents were to choose labor effort, for example). Analogously, mandatory spending on entitlements e is equivalent to considering the consumption of the poor as a status-quo value, since $c_P = y_P + e$. Because of this, we can re-define our benchmark case as one in which we have mandatory spending in *private goods* (and refer to it as a c-rule) and the one in Bowen et al. (2014) as one in which there is mandatory spending on public goods (and refer to theirs as a g-rule). In terms of the bargaining game, under the g-rule the state space is given by $\mathbf{s} = \bar{g}$, whereas there is no predetermined value of e (e.g. it is equal to zero unless policymakers choose $e > 0$) and taxes are determined residually from the government budget constraint $\tau = e + g$. Under the c-rule, the state can be written as $\mathbf{s} = \{\bar{c}_R, \bar{c}_P\}$ (which is a simple re-normalization relative to our benchmark $\{\bar{e}, \bar{\tau}\}$ in the main text).

A.8.1 Quasi-linear utility

We present our results for a simple case where the utility function of the parties is quasi-linear

$$u(c_{i,t}, g_t) = c_{i,t} + \theta\ln(g_t), \quad \text{for } t = \{1, 2\}.$$

We are deviating from the log-log benchmark specification in order to make the comparison with Bowen et al. (2014) more transparent. The mathematical derivation of second period allocations and first period welfare is contained in the Online Appendix. Here, we just want to emphasize how first-period allocations are affected by the type of budget rule. For reference, note that with quasi-linear utility the Samuelson-level of public good provision is constant and equal to

$$g^* = 2\theta.$$

In addition, the level of g when there are no budget rules is $g^D = \theta$ in this case. That is, when all spending is discretionary, the proposer sets the private marginal cost of providing g to its private marginal benefit, θ. The latter is smaller than the social marginal benefit of 2θ, implying that the public goods are under provided when there are no rules.

To study how budget rules affect allocations, we depart from a situation where the proposer is unconstrained (e.g. the acceptance constraint is slack) in the first period (consistently with in the main text). This allows us to have closed-form solutions that can provide the necessary intuition for our comparison between spending rules that affect public good provision and spending rules that affect private goods.

Under a c-rule, it is optimal for an R-type proposer to set in $t = 1$

$$1 = \frac{\theta}{g_1} - \beta(1-q)W_R'(c_{R,1}).$$

Since $W_R'(c_{R,1}) \geq 0$, mandatory spending on private goods create a negative wedge on public good provision. After some manipulations, we find that

$$\text{c-rule:} \quad g_1 = \frac{\theta}{1+\beta(1-q)},$$

which is increasing in q. Under a g-rule (e.g. mandatory spending on public goods), proposer R sets

$$1 = \frac{\theta}{g_1} + \beta q V_R'(g_1) + \beta(1-q)W_R'(g_1)$$

Since $W_R'(g_1) \geq 0$ and $V_R'(g_1) \leq 0$, the total wedge on public good provision depends on the relative size of $\beta q V_R'(g_1)$ and $\beta(1-q)W_R'(g_1)$. As shown in the Online Appendix,

$$\text{g-rule:} \quad g_1 = \frac{(1+\beta)\theta}{1+\beta q},$$

with g_1 *decreasing* in q.

Mandatory spending rules create a positive wedge on the provision of the good that is being made mandatory. Under a *c-rule*, it is more beneficial to provide relatively more private consumption because this ensures a strong bargaining power in the second period. Because of this, g is under-provided: $g_1 < g^*$. As q increases, and the current proposer is more likely to remain in power, the benefits of such distortions decrease. Therefore, g_1 gets closer to the efficient level g^* when q rises under a *c-rule*. The left panel of Figure (16) illustrates this (see dashed blue line). In the right panel we can see that private consumption decreases with q instead. Note, however, that even when q is large, $g_1 \leq g^D < g^*$.

In the case of the *g-rule*, the proposer has incentives to provide relatively more public goods. The higher the q, the lower is the benefit of distorting public good provision upwards. The dotted

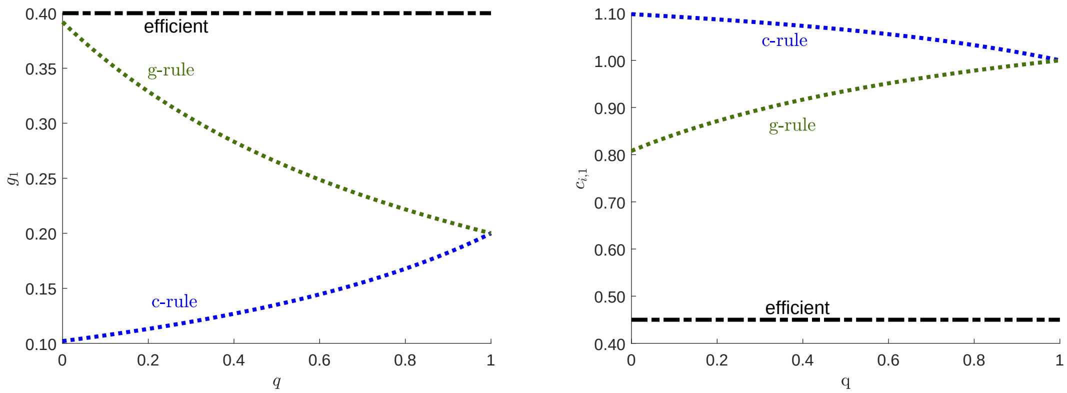

Figure 16: First-period allocations for all $q's$. $Y = 1.3$, $\bar{x} = 0.1$, $\beta = 0.96$ and $\theta = 0.2$.

green line in the left panel of Figure (16) illustrates this. Private consumption, on the other hand, rises with q. Consider the extreme case where $\beta = 1$ (perfectly patient society) and $q = 0$ (deterministic turnover). In such case, the $g - rule$ would restore optimality in period 1, with $g_1 = 2\theta$. This is in line with the results in Bowen et al. (2014), where mandatory spending rules tend to be beneficial for society: as long as the rule is set on the pure public good, it will push the proposer towards providing more of it. We find, however, that a $c - rule$ pushes to proposer away from the efficient value when turnover is high.

The impact of these budget rules on welfare depends on the identity of the party and on the level of political turnover. In this specific numerical example, the *g-rule* is always be better than the *c-rule* for society as a whole. This is clear in Figure (17), which shows the *g-rule* (bar green) always above the no-budget-rules line (dotted magenta) and the *c-rule* (dashed blue) for all values of q.[26]

A.8.2 The role of concavity

Consider the case where utility is logarithmic in private and public goods.

$$u(c_{i,t}, g_t) = \ln(c_{i,t}) + \theta \ln(g_t), \quad \text{for } t = \{1, 2\}.$$

This is the benchmark case analyzed in the paper, but under a generic value of θ. The allocations arising under budget rules on taxes and entitlements are characterized in the Online Appendix, Section 2.

Concavity is important for two reasons. First, because it affects the level of distortions in the provision of private in public goods deferentially for alternative rules. Second, because agents prefer scenarios exhibiting low volatility of private good consumption. These two forces are discussed next.

Concavity and distortions, a first-moment effect: In the first period, under a c-rule, the proposer sets

[26]Our numerical inspection suggests the same result is true for the case of log-log utility, i.e., the *g-rule* is always better than the *c-rule* when we start with a slack acceptance constraint. This is intuitive, since with a slack acceptance constraint the proposer has the advantage of distorting allocations to her favor in the second period. If the proposer starts from a binding constraint, however, it is easy to see numerically that the *c-rule* can be better than the *g-rule*.

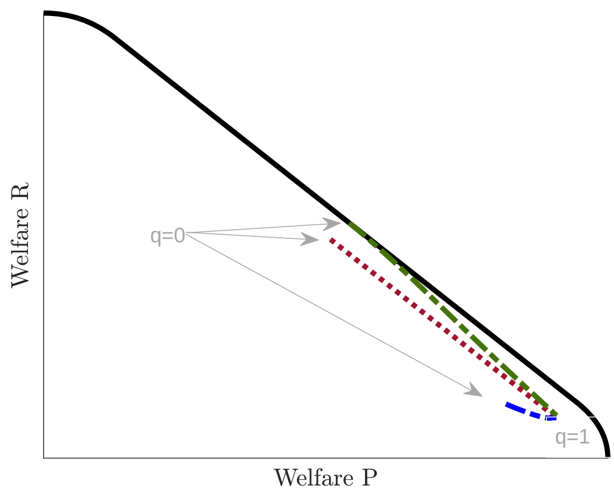

Figure 17: Pareto frontier (black), *c-rule* (dashed blue), *g-rule* (bar green), no rules (dotted red) for all $q's$. $Y = 1.3$, $\bar{x} = 0.1$, $\beta = 0.96$ and $\theta = 0.2$.

$$\frac{1}{c_{P,1}} + \beta(1-q)W'_P(c_{P,1}) = \frac{\theta}{g_1}.$$

Under a g-rule, it sets

$$\frac{1}{c_{P,1}} = \frac{\theta}{g_1} + \beta\left\{qV'_P(g_1) + (1-q)W'_P(g_1)\right\}.$$

As in the quasi-linear case, mandatory spending on different goods creates distortions on the first-period's proposer optimal conditions. These induce the policymaker to provide relatively more of the good subject to the budget rule. With concave utility in both goods, the marginal utility of private consumption is not always 1 (as it was in the linear case); it now depends on the value of c. The following table summarizes public good allocations for the *c-rule* and the *g-rule*, in the quasi-linear and concave utility cases. Optimal and no-rules solutions are also provided for reference.

g_1 under	**Quasi-linear**	**Concave**
Optimal	2θ	$\frac{\theta Y}{1+\theta}$
No rules	$g_l^D = \theta$	$g_c^D = \frac{\theta(Y-\bar{x})}{1+\theta}$
g-rule	$\frac{(1+\beta)\theta}{1+\beta q} > g_l^D$	g_c^D
c-rule	$\frac{\theta}{1+\beta(1-q)} < g_l^D$	$\frac{\theta(Y-\bar{x})}{1+\theta+\beta(1-q)} < g_c^D$

Table 4: Provision of g_1 under alternative budget arrangements and utility functions.

The level of distortions in the provision of public goods, then, is different qualitatively and quantitatively. For example, under a g-rule public good provision is closer to optimal (e.g., an improvement relative to the no-rules scenario) only when utility is quasi-linear.

Concavity and volatility, a second-moment effect: In addition to whether the rule induces more or less provision of public goods, the volatility of private consumption allocations over time matters when utility is concave, but it does not when utility is linear in c. Because of concavity in private goods' utility, agents prefer smooth consumption profiles.

The easiest way to understand how volatility affects welfare is by looking at the solution of a two period model under no budget rules, assuming that one party is the proposer in the first period and the other party is the proposer in the second period. For illustration, suppose additionally that $\beta = 1$. In that case, proposer i sets consumption to the opposition at its lowest value, $c_j = \bar{x}$ at each point in time and its own level of consumption is $c_{i,i}^D > \bar{x}$ for every t. Public good provision is g^D regardless of the identity of the proposer.

Compare the proposer's welfare under two alternative sequences of private consumption $\mathbf{a_1}$ and $\mathbf{a}_2$, defined by $\mathbf{a}_1 = \{c_{i,i}^D, \bar{x}\}$ and $\mathbf{a}_2 = \{\tilde{x}, \tilde{x}\}$ with $\tilde{x} = \frac{c_{i,i}^D + \bar{x}}{2}$. The first value denotes consumption in the first period and the second value consumption in the second period. These two allocations give the same welfare under linear utility, as lifetime utility becomes $c_{i,i}^D + \bar{x}$ in both cases. Under concave utility, instead, the proposer would be better off under $\mathbf{a_2}$ than it would be under $\mathbf{a_1}$, because the former involves a smoother consumption profile. A similar argument holds for the opposition, with $\mathbf{a}_1 = \{\bar{x}, c_{j,j}^D\}$. Budget rules affect how volatile sequences of c and g are in the bargaining equilibrium. More volatile sequences in c generate welfare losses only if $u(c)$ is concave. Therefore, using c as a bargaining chip to move the status-quo value of g (e.g. under a g-rule) is "cheaper" when utility is quasi-linear than it is when utility is concave in both goods.

A.9 Alternative budget rules

In this appendix, we define the maximization problem of a proposer under alternative budget rules considered in the paper. Under a P proposer,

$$\max_{\{\tau,e,g\}} u(c_P) + \theta u(g) + \beta \Big\{ q V_P(\pi_P) + (1-q) W_P(\pi_P) \Big\} \tag{40}$$

where $\mathbf{s}' = \pi_P$, subject to eqs. (1)-(4), and the *acceptance constraint*

$$\begin{aligned} u(c_R) + \theta u(g) + \beta \Big\{ (1-q) V_R(\pi_P) + q W_R(\pi_P) \Big\} \quad &\geq \\ u(\overline{c_R}) + \theta u(\overline{g_d}) + \beta \Big\{ (1-q) V_R(\mathbf{s}) + q W_R(\mathbf{s}) \Big\} & \end{aligned} \tag{41}$$

The bottom part of the equation denotes the payoff from keeping the status quo. Here $\overline{g_d}$ represents public good provision and $\overline{c_R}$ the level of consumption of the rich when the proposal is rejected. The value of the state variable next period π_P, as well as the default values when no agreement is made (bottom line of last equation), are determined by the specific budget rule. They are summarized in Table 5, where we used the fact that $y_P = \overline{x_g}$.

Notice that even though it may seem that resources are "wasted" under some of the rules (e.g. GBC is slack), in equilibrium, we find that it is not optimal to choose pairs where the government budget constraint does not hold. In other words, in the ergodic set, it is always the case that state variables are such that the GBC holds with equality.

A.10 Computation with Taste Shocks

A well-known issue in dynamic legislative bargaining games with endogenous status quo is that standard algorithms are not always successful in computing Markov Perfect Equilibria. This paper

Endogenous status quo: **s**	τ&e-rule $\{\overline{\tau},\overline{e}\}$	τ-rule $\{\overline{\tau}\}$	e-rule $\{\overline{e}\}$	g-rule $\{\overline{g}\}$
Status-quo values: π				
Public Goods	$\overline{x_g}$	$\overline{x_g}$	$\overline{x_g}$	$\overline{g}$
Entitlements	$\overline{e}$	0	$\overline{e}$	0
Taxes	$\overline{\tau}$	$\overline{\tau}$	$\overline{x_g}+\overline{e}$	$\overline{g}$
Cons. Poor: $\overline{c_P}$	$\overline{x_g}+\overline{e}$	$\overline{x_g}$	$\overline{x_g}+\overline{e}$	$\overline{x_g}$
Cons. Rich: $\overline{c_R}$	$y_R-\overline{\tau}$	$y_R-\overline{\tau}$	$y_R-(\overline{x_g}+\overline{e})$	$y_R-\overline{g}$

Table 5: Alternative budget rules (note that $y_P = \overline{x_g}$)

is no exception, as standard value-function iteration procedures do not converge for arbitrary calibrations of the model. It is worth noticing that the convergence problems are not caused by the bounds of taxes and entitlement or the specific form of the utility function. We have experimented with quadratic utility and no minimum consumption and the issue remains (details available upon request). The fact that the computation of models in this class is notoriously challenging has been documented by Duggan and Kalandrakis (2012), Martin (2009), Chatterjee and Eyigungor (2012), and others. A common strategy, also adopted here, is to slightly perturb the choices of the proposing agent through the introduction of small, independent and identically distributed shocks. These shocks may apply to fundamentals, as in Chatterjee and Eyigungor (2012), or to the agent's payoff directly, as in Eyigungor and Chatterjee (2019) or Dvorkin et al. (2021). We follow Gordon (2019)[27] and use the functional forms and assumptions employed with discrete choice methods. The resulting randomization over options with payoff of comparable value greatly eases the computation of the model, induces smooth value functions and policy functions, and induces near-monotone convergence via standard value function iteration[28].

We perturb the proposer's problem by augmenting it with choice-specific taste shocks, distributed Gumbel (Extreme Value Type I), as commonly employed with discrete choice methods, e.g. Rust (1987). To simplify notation, let the status quo be given by $\overline{s} = \langle \overline{\tau}, \overline{e} \rangle$ and any potential choice denoted by $s = \langle \tau, e, g \rangle$. As before, $c_P = y_P + e$ and $c_R = yR - \tau$, so we suppress any explicit dependency of consumption levels on the proposal s.

Define the acceptance set for agent $j \in \{P, R\}$ as

$$\mathcal{A}_j(\overline{s}) = \left\{ s \in \mathbb{S} \,\middle|\, (1-\beta)u(c_j, g) + \beta\left[(1-q)V_j(s) + qW_j(s)\right] \geq K(\overline{s}) \right\}. \tag{42}$$

We write the value to the proposer $i \in \{P, R\}$ from proposing s, net of taste shocks, as

$$\mathcal{J}_i(\overline{s}, s) = \begin{cases} (1-\beta)u(c_i, g) + \beta\left\{qV_i(s) + (1-q)W_i(s)\right\}, & \text{if } s \in \mathcal{A}_j(\overline{s}) \\ -\infty, & \text{otherwise} \end{cases} \tag{43}$$

and record the greatest value over s by

$$\overline{\mathcal{J}}_i(\overline{s}) = \max_{s \in \mathbb{S}} \mathcal{J}_i(\overline{s}, s) \tag{44}$$

[27] Related, recent applications to fiscal policy and sovereign default include Dvorkin et al. (2021), Mihalache (2020), and Arellano et al. (2020).

[28] The method requires the introduction of a parameter governing the importance of these taste shocks for the agent's behavior. We set this parameter to the smallest value consistent with convergence within 1,000 iterations, at $\rho = 5e^{-4}$.

The value to the proposer—given realized iid taste shocks $\{\varepsilon_s\}_s$, one for each possible proposal—is

$$\mathcal{V}_i(\bar{s}, \{\varepsilon_s\}_s) = \max_{s \in \mathbb{S}} \{\mathcal{J}_i(\bar{s}, s) + \rho\, \varepsilon_s\}. \tag{45}$$

Following the standard proofs for discrete choice methods, e.g. McFadden (1973), it can be shown that ex-ante, before taste shocks are realized, the probability of choosing a particular option $\hat{s}$ is given by

$$\Pr(s = \hat{s}|\bar{s}) = \frac{\exp\left[\mathcal{J}_i(\bar{s}, \hat{s})/\rho\right]}{\sum_z \exp\left[\mathcal{J}_i(\bar{s}, z)/\rho\right]} = \frac{\exp\left[\left(\mathcal{J}_i(\bar{s}, \hat{s}) - \overline{\mathcal{J}}_i(\bar{s})\right)/\rho\right]}{\sum_z \exp\left[\left(\mathcal{J}_i(\bar{s}, z) - \overline{\mathcal{J}}_i(\bar{s})\right)/\rho\right]} \tag{46}$$

and the expected value to the proposer, before observing the taste shocks, is

$$V_i(\bar{s}) = \mathbb{E}_{\{\varepsilon_s\}_s}\left\{\mathcal{V}_i(\bar{s}, \{\varepsilon_s\}_s)\right\} = \overline{\mathcal{J}}_i(\bar{s}) + \rho \ln\left\{\sum_z \exp\left[\left(\mathcal{J}_i(\bar{s}, z) - \overline{\mathcal{J}}_i(\bar{s})\right)/\rho\right]\right\} \tag{47}$$

while the value of the agent receiving the proposal is

$$W_j(\bar{s}) = \sum_z \left\{\Pr(s = z|\bar{s})\left[u(c_j, g) + \beta\left((1-q)V_j(z) + qW_j(z)\right)\right]\right\}. \tag{48}$$

We remark briefly on key properties of the choice probabilities and of the solution. First, the probability of choosing s is strictly increasing in the value net of taste shocks for s, $\mathcal{J}_i(\bar{s}, s)$, so that better options are picked with higher probability. Second, given our use of the acceptance set, all s that would not be accepted are proposed to probability zero. Third, the mean level of the Gumbel tastes shocks is non-zero, yet this does not alter choice probabilities: what matters for the likelihood of choosing s over e.g. z is the difference between ε_s and ε_z, which is Logistic(0, 1), as well as values net of taste shocks. We can and do remove the mean contribution of taste shocks to welfare by subtracting the Euler-Mascheroni constant times ρ. The parameter ρ scales the importance of the taste shocks for the proposal decision. If we take the value $\rho \to 0$, tastes shocks no longer play a role and the underlying best option is picked with probability 1. In turn, for arbitrarily high ρ values, all members of the acceptance set would be proposed with equal probability. Small values of ρ, such as our $5e^{-4}$ are sufficient to induce convergence when iterating over V_i and W_j.