Non Linear Dividend Taxation and Shareholder Disagreement

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Abstract

We discuss an unintended side-effect of progressive dividend taxation in environments with heterogeneous households, that it leads to disagreement among shareholders over the level of investment that the firm should undertake. To resolve this disagreement, we assume that shareholders vote on the investment level, so that in equilibrium investment is consistent with the wishes of the "median shareholder," the agent which holds the "median share," with the property that 50% of shares are held by agents who'd prefer weakly higher investment, with the rest held by those preferring lower levels. Over time the median share changes hands endogenously, due to idiosyncratic shocks and endogenous capital dynamics. One important contribution of the paper is to formulate and solve the model recursively. We characterize the requirements of price-taking behavior in the stock market and construct price conjecture that support competitive, forward-looking investment preferences. We find that shareholder disagreement leads to endogenous fluctuations in capital, the firm's size, even in the absence of aggregate uncertainty, whenever the identity of the median shareholder changes, increasing the volatility of investment and stock prices, with potentially negative consequences for risk sharing.

1 Introduction

Nonlinear dividend taxation is a common feature of tax codes, since dividend payments are typically taxed at the same rate as regular income, which in turn is subject to progressive taxation. We show that there is a novel and unintended effect of such dividend taxation in settings with heterogeneous households, it leads to disagreement among shareholder over the level of investment that the firm should implement. We consider a simple one-share-one-vote mechanism to resolve this disagreement, resulting in investment policies in line with the

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Median Voter Theorem of Downs (1957) and the following seminar literature. The one-dimensional issue of how much to grow the firm leads naturally to a characterization based on the proposal of the "median shareholder," the only one which cannot be overruled by a majority of shareholders. We set to explore the rich positive and normative implications of voting among disagreeing shareholders, for risk-sharing, asset pricing and investment decisions.

Whereas the issue of shareholder disagreement has been mostly assumed away in the macroeconomic literature with incomplete markets, often by modelling the firm as a static entity with heterogeneous households that own and rent out the capital to the firm, this issue has received much more attention in the theoretical literature, starting from the seminal paper of Diamond (1967). While the macroeconomic work on heterogeneous households and incomplete markets typically assumes constant returns to scale production technologies that use as inputs capital and labor, the theoretical literature typically considers models in which capital is the only input in production, subject to decreasing returns to scale (see Dreze, 1985, or Magill and Quinzii, 1996, for a review of this work). In such a setting, Ekern and Wilson (1974) show that shareholder unanimity will arise if the firm's vector of dividends is spanned by the payoffs of existing securities.

In an important contribution to the macroeconomic treatment of shareholder disagreement, Coen-Pirani and Carceles-Poveda (2010) show that the Ekern and Wilson (1974) spanning condition is implied by the constant returns to scale assumption made in typical macroeconomic settings. This, together with the assumption of perfect competition, in the sense that a firm's shareholders take share prices as given, leads to shareholder unanimity in the standard model with incomplete markets. In essence, the assumption that shareholders are price-takers requires that shareholders use *price conjectures* to evaluate the effect of alternative investment choices on the stock price in a way that does not internalize the effect of these choices on their own pricing kernel. Recently, Bisin et al (2017) have shown that one can specify price conjectures that also result in shareholder unanimity over the investment plan of the firm, and they use this model to analyze the impact of agency frictions in a setup with heterogeneous households and incomplete markets.

We show that incorporating nonlinear dividend taxation in an otherwise standard heterogeneous households setup breaks the unanimity result and leads to shareholder disagreement, even if the right price conjectures are used. Moreover, disagreement arises even when markets are complete. As discussed above, to determine the economy-wide optimal capital stock, we use a median voter result which requires that the level of capital chosen by the firm has the property that 50% of shares are held by shareholders who want a weakly lower one while the other 50% is held by shareholders who want a weakly higher one. We first illustrate how to implement this rule in a stylized three period model with two shareholder types, agent-specific dividend taxes, perfectly negatively correlated endowments and no uncertainty. In such a setting, the median voter is relatively easy to identify, since the median share is held by whichever agent happens to hold over 50% of the shares. Subsequently, the model is extended to a two agent infinite horizon economy with uncertainty, and ongoing work aims to extend these results to a model with a continuum of agents as in Aiyagari (1994), subject to progressive dividend taxes. An important contribution of the paper is to formulate and solve the model recursively. To our knowledge, this is the first paper that does this in the presence of shareholder disagreement, taking into account that the effective investment decision-maker changes over time, depending on the endogenous choice of shareholdings.

As standard in two-agent models, the joint distribution of wealth and productivity affects the law of motion of the aggregate capital stock due to the presence of incomplete markets and binding borrowing constraints. In particular, a higher joint dispersion of individual shareholdings and income shocks will lead to a significantly higher capital accumulation compared to the case of little or no dispersion. Intuitively, when "poor" agents are close to the no-short-selling constraint, market clearing requires a downwards adjustment of the return on capital (so that "rich" agents do not want to save as much), which can be achieved by expanding aggregate capital. Whereas this mechanism alone leads to fluctuations in the aggregate capital even in the absence of aggregate uncertainty, we show that shareholder disagreement provides an additional novel source of endogenous fluctuations in capital. Since agents are subject to different dividend tax rates, each type's desired investment levels gravitate towards different "steady states" so that, even in the absence of binding borrowing constraints, the implemented investment and capital fluctuate over time, based on the identity of the median shareholder.

The previous result has two important implications. First, disagreement among shareholders increases the volatility of investment and stock prices. Second, in the presence of progressive taxation, shareholder disagreement can further boost the accumulation of capital which typically arises in models with incomplete markets due to precautionary savings motives. In our environment, the shareholder subjected to the highest dividend tax rate will have the highest desired level of capital, since a higher tax rate implies a higher marginal rate of substitution and hence a stronger desire to transfer resources into the next period. At the same time, the majority/median shareholder in our model is often the one that receives the higher endowment shock. Thus, in the presence of progressive taxation, we expect the shareholder with the highest tax to typically end up making the investment decision, leading to an overall higher capital level.

The paper is organized as follows. Section two presents a two-period analytical example without uncertainty, to illustrate how type-specific taxation can generate disagreement among the firm's shareholders. Section three generalizes the previous model to illustrate how the equilibrium level of investment can be determined endogenously, via a shareholder voting in a multi-period setting. Moreover, it also illustrates the formulation of price conjectures consistent with a competitive market in shares, to allow for the evaluation of counterfactual investment choices on the market value of the firm. Section four extends the model to an infinite horizon and introduces uncertainty, to show how to formulate and solve recursively such an environment, and then discusses the results in the associated numerical solution. Section 5 summarizes and concludes.

2 Dividend Taxes and Shareholder Disagreement

In this section, we present a simple two period analytical example to illustrate how type specific taxation can generate disagreement among the firm's shareholders.

Time is discrete and indexed by t = 0, 1. The economy is populated by two households that are indexed by $i = 1, 2$ and by a continuum of identical firms. There is no aggregate or individual uncertainty. There is no production at period 0 but firms produce a good in period 1 with the production function $AF(K) = AK$, where K is the physical capital input that is invested in period $t = 0$ and A is a fixed technology parameter. Each period, firms pay a per share dividend of d_t to the firm shareholders. The timing of the consumer decisions is as follows. At the beginning of time 0, consumers receive a fixed endowment e_0^i, with $\sum_{i=1}^{2} e_0^i = 1$, and dividend payments net of taxes $d_0\theta_0^i\left(1-\tau^i\right)$ from their initial share ownership in the firm θ_0^i, with $\sum_{i=1}^{2}\theta_0^i = 1$ and $d_0 = -K$ since there is no production at $t = 0$. When the stock market opens, consumers can change their portfolio by buying or selling their initial shares at the (ex-dividend) price p_0. At the beginning of period 1, the consumer receives a fixed endowment e_1^i, with $\sum_{i=1}^{2} e_1^i = 1$, and dividends $d_1\theta_1^i$, where $d_1 = AK$ and θ_1^i denotes the final final share of the firm held by consumer i after trading, with $\sum_{i=1}^{2}\theta_1^i = 1$. For simplicity, we assume that shareholders do not pay dividend taxes in period $t = 1$.

In what follows, we compute the competitive equilibrium for a given investment level K. Given (K, p_0), consumers solve the following problem:

$$\max_{\left\{c_0^i, c_1^i, \theta_1^i\right\}} \log\left(c_0^i\right) + \beta\log\left(c_1^i\right) \text{ s.t.}$$

$$\begin{aligned} c_0^i &= e_0^i + p_0\theta_0^i + \theta_0^i d_0\left(1-\tau^i\right) - p_0\theta_1^i \\ c_1^i &= d_1\theta_1^i + e_1^i \\ d_0 &= -K,\ d_1 = AK \end{aligned}$$

where β is the discount factor. The optimality conditions imply that the stock price is given by:

$$p_0 = \frac{\beta c_0^i}{c_1^i} d_1 \text{ for } i = 1, 2 \tag{1}$$

Using the optimality condition above, we can derive the asset trade policy as a function of (K, p_0):

$$\theta_1^i\left(K, p_0\right) = \frac{\beta AK\left[e_0^i + \theta_0^i\left(p_0 - K\left(1-\tau^i\right)\right)\right] - p_0 e_1^i}{p_0 AK\left(1+\beta\right)} \tag{2}$$

Moreover, using the asset market clearing condition, $\theta_1^1\left(K, p_0\right) + \theta_1^2\left(K, p_0\right) = 1$, we can obtain the price capital mapping, namely, the function that determines the combinations of capital and stock prices that are compatible with market clearing:

$$p_0^* (K) = \frac{\beta \left[1 - K \sum_{i=1}^{2} \theta_0^i \left(1 - \tau^i\right)\right]}{AK + 1} AK = \frac{\beta C_0}{C_1} d_1 \tag{3}$$

As reflected by the previous condition, the price capital mapping is just equal to the aggregate marginal rate of substitution times the period 1 dividend payments. Using this equation, we can substitute $p_0^* (K)$ into $\theta_1^i (K, p_0)$ and obtain the corresponding share allocation $\theta_1^{i*} (K)$ as a function of K only, and we can then use the budget constraints to obtain the corresponding consumption allocations $c_0^{i*} (K)$ and $c_1^{i*} (K)$ that are compatible with equilibrium given K.

So far, the previous allocations have been determined for a given level of K. Next, we determine each shareholder's preferred capital k^i to investigate whether there is agreement among the different shareholders. To do this, denote the value function corresponding to the allocations above as $W^i (K)$. The preferred capital of shareholder i is the one that maximizes this function with respect to capital, namely, for each $i = 1, 2$:

$$k^i = \arg\max_K W^i (K) = \arg\max_K \left[\log \left(c_0^{i*}\right) + \beta \log \left(c_1^{i*}\right)\right]$$

Clearly, a change in capital affects the shareholders' consumption both via dividends (d_0, d_1) and stock prices p_0[1]. Whereas it is straightforward for households to calculate the effect of changes in capital on the dividend payments, it is not clear what households should use to calculate the effect of capital changes on the stock market price p_0. Following the literature on shareholder disagreement[2], we introduce individual price conjectures $p_0^i (K)$ that each households can use to calculate how changes in capital affect stock prices. It is important to note that these conjectures cannot be chosen arbitrarily. In fact, they have to satisfy two very important conditions: (i) they have to ensure that households are price takers, in the sense that they don't internalize the effect of changes in capital on the pricing kernel or their own marginal rates of substitution; (iia) if shareholders agree on the level of capital that the firm should invest, their conjectures will have to satisfy the consistency condition:

$$p_0^i (K) = p_0 \tag{4}$$

(iib) if shareholders disagree on the level of investment, only the conjecture of the household that determines the investment level will have to satisfy the consistency condition. The following price conjectures satisfy the above conditions:

$$p_0^i (K) = b^i d_1 \tag{5}$$

where b^i is taken as given by the households when they determine their desired capital stock, but will be determined endogenously when we solve the model in

[1] As we will show later, the effect of a change in capital on the asset trades θ_1^i will cancel out by the optimality condition in (1).

[2] See XXX

order to satisfy the consistency condition in (4). Given this price conjecture, the effect of a change in capital on the stock price can be expressed as:

$$\frac{\partial p_0^i\left(K\right)}{\partial K} = b^i \frac{\partial d_1\left(K\right)}{\partial K}$$

We are now ready to determine the shareholder's preferred capital. The optimality condition with respect to capital for each shareholder is given by:

$$\frac{1}{c_0^{i*}} \frac{\partial c_0^{i*}}{\partial K} + \beta \frac{1}{c_1^{i*}} \frac{\partial c_1^{i*}}{\partial K} = 0 \rightarrow$$

$$\begin{aligned} 0 &= \frac{1}{c_0^{i*}} \left[\frac{\partial p_0^i\left(K\right)}{\partial K} \theta_0^i + \theta_0^i \frac{\partial d_0\left(K\right)}{\partial K} \left(1 - \tau^i\right) \right] + \left[\beta \frac{1}{c_1^{i*}} \frac{\partial d_1\left(K\right)}{\partial K} - \frac{1}{c_0^{i*}} \frac{\partial p_0^i\left(K\right)}{\partial K} \right] \theta_1^i \\ &\quad + \left[\beta \frac{1}{c_1^{i*}} d_1 - \frac{1}{c_0^{i*}} p_0^i\left(K\right) \right] \frac{\partial \theta_1^i\left(K\right)}{\partial K} \end{aligned} \tag{6}$$

In the previous equation, the last term, which reflects the effects of a change in capital on the share trades, cancel out by the optimality condition in (1). In addition, the consistency condition implies that $b^i = \frac{\beta c_0^i}{c_1^i}$ and the second term also cancels out. Given this, the preferred capital for each shareholder satisfies the following condition:

$$\frac{1}{c_0^{i*}} \theta_0^i \left[\frac{\beta c_0^{i*}}{c_1^i} A - \left(1 - \tau^i\right) \right] = \frac{1}{c_0^{i*}} \theta_0^i \left[\frac{p_0}{K} - \left(1 - \tau^i\right) \right] = 0$$

Several important observations emerge by looking at the previous equation. The first square bracket reflects that the desired investment level of shareholder i satisfies the standing Euler condition that would arise in a model in which the firm makes the investment decisions by maximizing the firm value, with the marginal rate of substitution of shareholder i as a discount factor. Morever, the second square bracket clearly reflects that type specific dividend taxes will lead to shareholder disagreement with respect to the lavel of investment that the firm should implement. In particular, if $\tau^1 = \tau^2$, the previous condition is satisfied for $k^i = \frac{p_0}{(1-\tau)}$ and both shareholders agree on this level of investment. In contrast, if taxes differ across shareholders, the desired level of investment is $k^i = \frac{p_0}{1-\tau^i}$, reflecting that shareholders disagree on level of investment. In the next section, we generalize this model and discuss a mechanism that can be used to determine the level of investment.

3 A Finite Horizon Model with Disagreement

In this section, we present a generalization of the previous model to illustrate how the optimal level of investment can be determined via majority voting in a multiperiod setting with shareholder disagreement. Moreover, we also show

how we can define price conjectures that are consistent with a competitive equilibrium.

The model in this section is essentially a three period version of the one in the previous section with a more general firm technology $AF(K) = AK^{\alpha}$ and household utility function $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. Given (p_0, p_1) and (K_1, K_2), consumers solve the following problem:

$$\max_{\{c_0^i, c_1^i, c_2^i, \theta_1^i, \theta_2^i\}} u\left(c_0^i\right) + \beta u\left(c_1^i\right) + \beta^2 u\left(c_2^i\right) \text{ s.t.}$$

$$\begin{aligned} c_0^i &= e_0^i + p_0 \theta_0^i + \theta_0^i d_0 \left(1 - \tau^i\right) - p_0 \theta_1^i \\ c_1^i &= \left(p_1 + d_1 \left(1 - \tau^i\right)\right) \theta_1^i + e_1^i - p_1 \theta_2^i \\ c_2^i &= d_2 \left(1 - \tau_i\right) \theta_2^i + e_2^i \\ d_0 &= AK_0^{\alpha} - K_1, \; d_1 = AK_1^{\alpha} - K_2, \; d_2 = AK_2^{\alpha} \end{aligned} \tag{7}$$

which yields the following optimality conditions for each household:

$$p_0 u_{c_0^i} = \beta u_{c_1^i} \left[d_1 \left(1 - \tau_i\right) + p_1\right] \tag{8}$$

$$p_1 u_{c_1^i} = \beta u_{c_2^i} \left[d_2 \left(1 - \tau_i\right)\right] \tag{9}$$

where $u_{c_t^i}$ denotes the marginal utility of agent i at period t. The asset market clearing conditions that will determine the equilibrium prices are:

$$\sum_{i=1}^{2} \theta_t^i = 1 \text{ for } t = 1, 2. \tag{10}$$

We now determine the preferred capital of each shareholder. To do this, we construct their value functions and maximize them with respect to the two capital choices K_1 and K_2. The optimality conditions with respect to K_1 and K_2 are:

$$\frac{\partial W^i}{\partial K_1} = u_{c_0^i} \frac{\partial c_0^i}{\partial K_1} + \beta u_{c_1^i} \frac{\partial c_1^i}{\partial K_1} + \beta^2 u_{c_2^i} \frac{\partial c_2^i}{\partial K_1} = 0 \rightarrow$$

$$\begin{aligned} 0 = {} & \theta_0^i u_{c_0^i} \left(\frac{\partial p_0^i}{\partial K_1} + \frac{\partial d_0}{\partial K_1} \left(1 - \tau_i\right) \right) + \theta_1^i \left[-u_{c_0^i} \frac{\partial p_0^i}{\partial K_1} + \beta u_{c_1^i} \left(\frac{\partial p_1^i}{\partial K_1} + \frac{\partial d_1}{\partial K_1} \right) \left(1 - \tau_i\right) \right] \\ & + \theta_2^i \left[-\beta u_{c_1^i} \frac{\partial p_1^i}{\partial K_1} + \beta^2 u_{c_2^i} \left(\frac{\partial d_2}{\partial K_1} \left(1 - \tau_i\right) \right) \right] \end{aligned} \tag{11}$$

$$\frac{\partial W^i}{\partial K_2} = u_{c_0^i} \frac{\partial c_0^i}{\partial K_2} + \beta u_{c_1^i} \frac{\partial c_1^i}{\partial K_2} + \beta^2 u_{c_2^i} \frac{\partial c_2^i}{\partial K_2} = 0 \rightarrow$$

$$\begin{aligned} 0 = {} & \theta_0^i u_{c_0^i} \left(\frac{\partial p_0^i}{\partial K_2} + \frac{\partial d_0}{\partial K_2} \left(1 - \tau_i\right) \right) + \theta_1^i \left[-u_{c_0^i} \frac{\partial p_0^i}{\partial K_2} + \beta u_{c_1^i} \left(\frac{\partial p_1^i}{\partial K_2} + \frac{\partial d_1}{\partial K_2} \left(1 - \tau_i\right) \right) \right] \\ & + \theta_2^i \left[-\beta u_{c_1^i} \frac{\partial p_1^i}{\partial K_2} + \beta^2 u_{c_2^i} \left(\frac{\partial d_2}{\partial K_2} \left(1 - \tau_i\right) \right) \right] \end{aligned} \tag{12}$$

As before, it is easy for the households to determine the effect of capital changes on dividends. In particular, if we let $K = (K_1, K_2)$, we have:

$$\begin{cases} d_0(K) = AK_0^\alpha - K_1 \\ d_1(K) = AK_1^\alpha - K_2 \\ d_2(K) = AK_2^\alpha \end{cases} \Rightarrow \begin{cases} \frac{\partial d_0(K)}{\partial K_1} = -1,\ \frac{\partial d_0(K)}{\partial K_2} = 0 \\ \frac{\partial d_1(K)}{\partial K_1} = A\alpha K_1^{\alpha-1},\ \frac{\partial d_1(K)}{\partial K_2} = -1 \\ \frac{\partial d_2(K)}{\partial K_1} = 0,\ \frac{\partial d_2(K)}{\partial K_2} = A\alpha K_2^{\alpha-1} \end{cases} \tag{13}$$

However, households need to use price conjectures $p_t^i(K)$ for $t = 0, 1$ to infer how capital changes will affect stock prices. To be consistent with the equilibrium, we define the price conjectures as follows:

$$\begin{aligned} p_1^i(K) &= b_1 d_2 \\ p_0^i(K) &= b_{0,1} d_1 + b_{0,2} d_2 \end{aligned} \tag{14}$$

which implies that the effects of changes in capital on stock prices can be expressed as follows:

$$\begin{cases} p_1^i(K) = b_1 d_2 \\ p_0^i(K) = b_{0,1} d_1 + b_{0,2} d_2 \end{cases} \Rightarrow \begin{cases} \frac{\partial p_1^i(K)}{\partial K_1} = 0,\ \frac{\partial p_1^i(K)}{\partial K_2} = b_1^i A\alpha K_2^{\alpha-1} \\ \frac{\partial p_0^i(K)}{\partial K_1} = b_{0,1}^i A\alpha K_1^{\alpha-1},\ \frac{\partial p_0^i(K)}{\partial K_2} = -b_{0,1}^i + b_{0,2}^i A\alpha K_2^{\alpha-1} \end{cases} \tag{15}$$

Clearly, consistency requires:

$$b_1 = \beta \frac{u_{c_2^i}}{u_{c_1^i}} (1 - \tau_i),\ b_{0,1} = \beta \frac{u_{c_1^i}}{u_{c_0^i}} (1 - \tau_i),\ b_{0,2} = \frac{b_{0,1} b_1}{1 - \tau_i} \tag{16}$$

In order to see how type specific taxes lead to shareholder disagreement in this environment, we can substitute the derivatives in (15) and (13) as well as the consistency conditions in (16) in the optimality conditions with respect to capital. If we do this, conditions in (11) and (12) become:

$$\theta_0^i u_{c_0^i} (1 - \tau_i) \left(\beta \frac{u_{c_1^i}}{u_{c_0^i}} A\alpha K_1^{\alpha-1} - 1 \right) = 0 \Rightarrow 1 = \beta \frac{u_{c_1^i}}{u_{c_0^i}} A\alpha K_1^{\alpha-1} \tag{17}$$

$$\theta_0^i \beta u_{c_1^i} (1 - \tau_i) \left(-1 + \beta \frac{u_{c_2^i}}{u_{c_1^i}} A\alpha K_2^{\alpha-1} \right) = 0 \Rightarrow 1 = \beta \frac{u_{c_2^i}}{u_{c_1^i}} A\alpha K_2^{\alpha-1} \tag{18}$$

Several important observations are worth noting. First, as before, the optimality condition for the preferred capital of shareholder i satisfies the standard capital Euler equation that would arise if the firm was chosing the investment level to maximize the value of the firm (or the present discounted value of dividends) using as a discount factor shareholder's i marginal rate of substitution. Second, since the marginal rates of substitution do not equalize across shareholders due to the fact that dividend taxes are type specific, it is clear that shareholders will disagree on the level of investment the firm should implement as long as taxes are different. Third, the previous optimality conditions for capital imply that $b_{0,1}$ and b_1 satisfy the following conditions, which are consistent with the competitive equilibrium:

$$\frac{b_{0,1}}{(1-\tau_i)} = \frac{1}{A\alpha K_1^{\alpha-1}}, \; \frac{b_1}{(1-\tau_i)} = \frac{1}{A\alpha K_2^{\alpha-1}}$$

In order to determine the optimal investment level in the presence of disagreement, we introduce a majority voting rule. In particular, let ID_t denote the identity of the investment decision maker in period t, with $ID_t = 1$ or 2. The majority voting rule in our two agent model then requires that $ID_t = i$ if $\theta_t^i \geq 0.5$. Using this rule, we can solve the model with the backward algorithm described below.

Solution Algorithm. To solve the model, we first go to period $t = 2$ and we discretize K_2 and θ_2^1, with $\theta_2^2 = 1 - \theta_2^1$. For each combination of the two variables $s = \left(K_2, \theta_2^1\right)$, we can calculate $d_2^*(s)$ and $c_2^{i*}(s)$ using (7). Note that the optimal levels for K_2 and θ_2^1 are actually chosen in period $t = 1$.

Next, we go to period $t = 1$ and we discretize K_1 and θ_1^1 (using the same grid as for K_2 and θ_2^i). For each point on the grid $s_1 = \left(K_1, \theta_1^1\right)$ with $\theta_1^2 = 1 - \theta_1^1$, we need to find $K_2^*(s_1)$, $p_1^*(s_1)$, $d_1^*(s_1)$, $\left\{\theta_2^{i*}(s_1), c_1^{i*}(s_1)\right\}_{i=1}^2$, with $s_2 = \left(K_2^*(s_1), \theta_2^{i*}(s_1)\right)$, by using (7), (8), (10) and the optimality condition for K_2, which can be expressed as[3]:

$$u^*_{c_1^{ID_1}}(s_1) = \beta u^*_{c_2^{ID_1}}(s_2)\, \alpha A \left(K_2^*(s_1)\right)^{\alpha-1}$$

where $ID_1 = 1$ if $\theta_1^1 \geq 0.5$ and $ID_1 = 2$ if $\theta_1^1 < 0.5$.

The last condition is the optimality condition for $K_2^*(s_1)$ from the investment decision maker in period 1, which is indexed by ID_1. As discussed earlier, the capital chosen is different depending on the identity of the decision maker because $\frac{u^*_{c_2^1}(s_2)}{u^*_{c_1^1}(s_1)} \neq \frac{u^*_{c_2^2}(s_2)}{u^*_{c_1^2}(s_1)}$.

Last, we go to period $t = 0$**.** We do not need a grid since $s_0 = \left(K_0, \theta_0^i\right)$ is a vector with just 2 given numbers. In this period, we need to find $K_1^*(s_0)$, $p_0^*(s_0)$, $\theta_1^{i*}(s_0)$, $d_0^*(s_0)$ and $c_0^{i*}(s_0)$ using again (7), (8) and (10), with $s_1 = \left(K_1^*(s_0), \theta_1^{i*}(s_0)\right)$. These can then be plugged into the functions above to find the values of all variables in periods $t = 1, 2$. To find $K_1^*(s_0)$, we need to use the following optimality condition with $s_2 = \left(K_2^*(s_1), \theta_2^{i*}(s_1)\right)$[4]:

$$1 = \beta \frac{u^*_{c_1^{ID_0}}(s_1)}{u^*_{c_0^{ID_0}}(s_0)} \alpha A \left(K_1^*(s_0)\right)^{\alpha-1} + \beta \frac{u^*_{c_1^{ID_0}}(s_1)}{u^*_{c_0^{ID_0}}(s_0)} \left[\beta \frac{u^*_{c_2^{ID_0}}(s_2)}{u^*_{c_1^{ID_0}}(s_1)} \alpha A \left(K_2^*(s_1)\right)^{\alpha-1} - 1\right] \frac{\partial K_2^*(s_1)}{\partial K_1^*(s_0)}$$

Notice that, if $ID_0 = ID_1$, the last term is zero by the optimal choice of K_2^* tomorrow, and the current capital K_1^* is chosen using the marginal rate of

[3] For a detailed derivation of this condition see Appendix A.
[4] Details of the derivations of this condition can be found in Appendix A.

substitution of ID_0. However, if $ID_0 \neq ID_1$, the last term does not cancel out because shareholders will try to influence K_2^* through the choice of K_1^* in order to bring the level of investment K_2 closer to their preferred level of capital.

Qualitative Findings. Since the model is very stylized, we don't provide the actual numerical results but just a summary of our main qualitative findings. First, in this model, households are heterogeneous if they have different endowments and/or initial shares in the firm. However, since markets are effectively complete in the absence of uncertainty, heterogeneity in endowments or initial shares will not affect the agents' capital choice as long as they face the same taxes. As discussed earlier, both shareholders agree on their desired capital stock in that case. However, in the presence of shareholder disagreement, the agent that is subject to a higher tax rate desires a higher level of capital. Intuitively, the fact that the asset returns have to equalize across households by the optimality condition implies that, in this model, the after tax marginal rates of substitution of the two households equalized in equilibrium. Given this, the agent that is subject to the higher tax rate will have a higher marginal rate of substitution. In turn this implies that the agent wants to transfer resources from today to next period, leading to a higher investment choice.

The last finding could have important implications in a model with uncertainty and incomplete markets. First, the agent that typically makes the investment decisions is the one with the higher endowment, since that agent is investing more in shares. Thus, in the presence of progressive taxation, the agent with higher taxes will typically end up making the investment decisions. In a model with uncertainty and market incompleteness, this can potentially enhance the overaccumulation of capital that results from precautionary savings. Second, if there is risk and endowments vary stochastically over time, the identity of the investment decision maker will change depending on the idiosyncratic shocks and this can potentially enhance the fluctuations in capital in the presence of progressive dividend taxation relative to the case with flat dividend taxes. In the following section, we present an infinite horizon economy with uncertainty in order to investigate these issues.

4 The Infinite Horizon Economy

In this section, we study a version of the previous model with an infinite horizon, idiosyncratic labor income risk and incomplete markets. Since there is type specific dividend taxation, there is shareholder disagreement on the level of investment, which we resolve using the majority rule used in the previous section.

4.1 The Model

The economy is populated by a representative firm and by two types of households, which are indexed by $i = 1, 2$, with a continuum of identical agents within each type. Households are initially endowed with equity shares in the firm which they can trade each period at price p_t but they cannot short sell A share in the firm entitles the owner to a dividend of d_t, which is subject to type specific dividend taxation. Apart from asset income, household i receives a stochastic labor endowment $e_{i,t}$ which follows a stationary Markov chain with transition matrix Π. As in the literature on two types, the shock is assumed to be perfectly negatively correlated across the types and, therefore, it is not purely idiosyncratic but, rather, redistributive[5]. Note that this also implies that the aggregate labor supply $N_t = \sum_{i=1}^{2} e_{i,t}$ is is constant, and therefore can be normalized to one without loss of generality. The individual labor income of agent i is equal to $w_t e_{i,t}$, where w_t is the aggregate wage rate paid by the representative firm.

Each period, the firm uses the aggregate capital K_t and the aggregate labor supply n_t to produce a good with the constant returns-to-scale technology $F(K,N) = K^{\alpha}N^{1-\alpha}$. Each period, the firm pays wages to the total labor employed and decides on the amount of aggregate investment K_{t+1}. The residual of gross profits (output net of labor payments) and investment is then paid as dividends to the firm equity owners.

Taken prices $\{p_t\}$ and the investment plan of the firm $\{K_t\}$ as given, households maximize:

$$\max_{\left\{c_t^i,\theta_{t+1}^i\right\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}\right) \text{ s.t.}$$

$$\begin{aligned}
c_t^i + p_t\theta_{i,t+1} &= \left(d_t\left(1-\tau_i\right) + p_t\right)\theta_{i,t} + w_t e_{i,t} \\
\theta_{i,t+1} &\geq 0 \\
d_t &= K_t^{\alpha} - w_t - K_{t+1} \\
w_t &= \left(1-\alpha\right) K_t^{\alpha}
\end{aligned}$$

where we have used the fact that $N_t = 1$. For each household i, the optimality conditions imply:

$$u'\left(c_{i,t}\right) \geq \beta\mathbb{E}_t\left[u'\left(c_{i,t+1}\right)\left(p_{t+1} + (1-\tau_1)d_{t+1}\right)\right]$$

with equality if $\theta_{i,t+1} > 0$. Finally, market clearing requires that $\sum_{i=1}^{2}\theta_{i,t+1} = 1$ for all t.

Since agents are subject to type specific dividend taxes they disagree on the level of investment K_{t+1} that the firm should implement. To resolve the disagreement, we assume that the agent with $\theta_{i,t+1} \geq 0.5$ at t is decision maker for K_{t+1} that period. In what follows, we describe in detail the algorithm that we use to solve the model resursively. To our knowledge, this is the first paper solving a dynamic model with disagreement among shareholders and we consider this an important contribution.

[5] We are currently working on an extension of the model to a continuum of agents.

4.2 Recursive Formulation and Solution Algorithm

To write the model and solution algorithm recursively, we first need to define the aggregate state variables. In the model, the aggregate state is $S = (K, \phi)$ where K is the aggregate capital and ϕ is the distribution of agents over shares and idiosyncratic shocks (θ, e). Since there are only two agents and their shocks are perfectly negatively correlated, the distribution can be approximated by $\phi = (\Theta, e)$, where Θ represents the shares of the agent who gets shock e, while the other agent's shares are equal to $1 - \Theta$. In sum, the aggregate state is given by $S = (K, \Theta, e)$. The law of motion (LOM) for the aggregate states is denoted by $G(K, \Theta, e)$ and is giveb by:

$$\begin{bmatrix} K' \\ \Theta' \end{bmatrix} = \begin{bmatrix} G_K(K,\Theta,e) \\ G_\Theta(K,\Theta,e) \end{bmatrix} = G(K,\Theta,e) \qquad : \qquad e' \sim \Pi(e'|e) \tag{19}$$

Step 1: Solving for the Competitive Equilibrium (CE) given an Aggregate LOM Our first step is to solve for the CE given an initial guess for the aggregate LOM $G(K, \Theta, e)$. To do this, we need guesses for the price function $p = P(K, \Theta, e)$ and the shareholding policies $\theta_i' = g_i^\theta(K, \Theta, e)$.

For each point in the aggregate state space (K, Θ, e) and each future aggregate state $K' = G_K(K, \Theta, e)$ and $\Theta' = G_\Theta(K, \Theta, e)$, given the future price function $p' = P(K', \Theta', e')$ and future shareholding policies $\theta_1''(e') = g_1^\theta(K', \Theta', e')$ and $\theta_2''(e') = g_2^\theta(K', \Theta', e')$, which are constructed from our initial guesses, we solve for (p, θ') in

$$\begin{cases} pu'(c_1) = \beta \mathbb{E}_{e'|e}\{u'(c_1'(e'))[p' + (1-\tau_1)d'(e')]\} \\ pu'(c_2) = \beta \mathbb{E}_{e'|e}\{u'(c_2'(e'))[p' + (1-\tau_2)d'(e')]\} \end{cases} \tag{20}$$

where:

$$\begin{cases} c_1 \equiv ew - p\theta' + [p + (1-\tau_1)d]\,\Theta \\ c_2 \equiv (1-e)\,w - p\left(1-\theta'\right) + [p + (1-\tau_2)d]\,(1-\Theta) \\ c_1'(e') \equiv e'w' - p'\theta_1''(e') + [p' + (1-\tau_1)d']\,\theta' \\ c_2'(e') \equiv (1-e')\,w' - p'\theta_2''(e') + [p' + (1-\tau_2)d']\left(1-\theta'\right). \\ d \equiv K^\alpha - w - K' \\ d'(e') \equiv (K')^\alpha - w' - G_K(K',\Theta',e') = \alpha\,(K')^\alpha - G_K(K',\Theta',e') \\ w = (1-\alpha)K^\alpha \\ w' = (1-\alpha){K'}^\alpha \end{cases}$$

The solution to this system of equations results in new shareholding policy and price functions, and we iterate and update these until convergence, namely, until $g_1^\theta(K, \Theta, e) \approx \theta'$ and $P(K, \Theta, e) \approx p$. The solution also delivers consumption policy functions $c_i = g_i^c(K, \Theta, e)$.

Step 2: Determining the Preferred Capital Our second step consists of determining the preferred capital $k_i = g_i^k(K, \Theta, e)$ for each shareholder. To do this, we need the consumption and shareholding policies $g_i^c(K, \Theta, e)$ and $g_i^\theta(K, \Theta, e)$ that we have computed in step 1. We also need guesses for the individual price conjectures $P_i^c(K, \Theta, e, k')$, which describe how an agent perceives the dependence of the stock price on off-equilibrium capital choices k', as well as for the value function $W_i(K, \Theta, e, k')$, which represents the continuation value for an agent of deviating to the investment level k'.

When computing the preferred capital, we make two important assumptions. First, we assume that agents do not reoptimize the shareholdings when considering deviations to different investment levels[6]. Second, we only consider a one time deviation in capital, implying that all the variables follow the equilibrium values the period after the deviation.

For each point in the aggregate state space (K, Θ, e) and each future aggregate state $K' = G_K(K, \Theta, e)$ and $\Theta' = G_\Theta(K, \Theta, e)$, we can compute the preferred capital for agent 1 by solving the following problem:

$$\begin{aligned} H_1(K, \Theta, e) &= \max_{k'} u(c_p) + \beta \mathbb{E}_{e'|e} W_1(K', \Theta', e', k') \\ &\text{s.t. } [K', : \Theta'] = G(K, \Theta, e) \\ &c_p = w(K)e + (1-\tau_1)(K^\alpha - w(K) - k')\Theta + \left[\Theta - g_1^\theta(K, \Theta, e)\right] P_1^C(K, \Theta, e, k') \\ w(K) &= (1-\alpha)K^\alpha \end{aligned} \tag{21}$$

where $H_1(K, \Theta, e)$ denotes the value for the agent of deviating to k' in state (K, Θ, e). The continuation value for agent 1 from a deviation to k is given by:

$$\begin{aligned} W_1(K, \Theta, e, k) &= u(c_p) + \beta \mathbb{E}_{e'|e} W_1(K', \Theta', e', k') \\ &\text{s.t. } [K', : \Theta'] = G(K, \Theta, e) \\ &k' = G_K(k, \Theta, e) \\ &c_p = w(K)e + (1-\tau_1)(k^\alpha - w(K) - k')\Theta + \left[\Theta - g_1^\theta(K, \Theta, e)\right] P_1^C(K, \Theta, e, k') \end{aligned} \tag{22}$$

and the price conjecture can be computed as follows:

$$\begin{aligned} P_1^C(K, \Theta, e, k') &= \mathbb{E}_{e'|e}\left\{B_1(K, \Theta, e, e')\left[(1-\tau_1')\left((k'^\alpha - w(K') - k'') + P_1^C(K', \Theta', e', k'')\right]\right\} \\ &\text{with } [K', : \Theta'] = G(K, \Theta, e) \text{ and } k'' = G_K(k', \Theta', e') \end{aligned} \tag{23}$$

where $B_1(K, \Theta, e, e')$ represents the marginal rate of substitution (MRS) of the agent, which is equal to:

$$B_1(K, \Theta, e, e') = \beta \frac{u'\left[g_1^c(K', \Theta', e')\right]}{u'\left[g_1^c(K, \Theta, e)\right]} \qquad \text{with } [K', : \Theta'] = G(K, \Theta, e)$$

[6] This assumption is common in the literature on incomplete markets and shareholder disagreement.

The solution to this problem is the preferred capital for agent 1, $k_1' = g_1^k(K, \Theta, e)$, and we can do an analogous calculation to obtain the preferred capital for the second agent $k_2' = g_2^k(K, \Theta, e)$.

Some important observations are worth noting. First, a capital deviation k' affects current consumption through the dividends and the price, an effect that is captured by the price conjecture $P_1^C(K, \Theta, e, k')$.In addition, it also affects the continuation value $W_1(K', \Theta', e', k')$ through k'. Note that the reason why we need tp have k as an argument in $W_i(K, \Theta, e, k)$ is to ensure that a deviation k' only affects dividends and not the MRS. As we see, the deviation affects the price conjecture today through tomorrow's dividend (k'^{α} as well as $k'' = G_K(k', \Theta', e')$), and tomorrow's price conjecture through $k'' = G_K(k', \Theta', e')$, but not through the MRS $B_1(K, \Theta, e, e')$.

Step 3: Determining the Capital Decision Maker and Updating the Aggregate LOM Our third step consists of determining the identity of the investment decision maker, which we denote by $ID(S)$, where $S = (K, \Theta, e)$ and hence the optimal level of investment can be expressed as $K' = g_{ID(S)}^k(K, \Theta, e)$, where $ID(S) = 1$ or 2. As discussed earlier, we assume that the investment decision maker is determined via a majority voting rule. In our two agent model, $ID(S) = 1$ if $g_1^{\theta}(K, \Theta, e) \geq 0.5$ and $ID(S) = 2$ otherwise. Using this and the individual policy functions computed in the previous steps, we can update the aggregate LOM using the consistency conditions as follows:

$$K' = G_K(K, \Theta, e) = \begin{cases} g_1^k(K, \Theta, e), & \text{if } g_1^{\theta}(K, \Theta, e) \geq 0.5 \\ g_2^k(K, \Theta, e), & \text{otherwise} \end{cases}$$

$$\Theta' = G_{\Theta}(K, \Theta, e) = g_1^{\theta}(K, \Theta, e)$$

Step 4: Updating the MRS, the Price Conjecture and the Value Functions In our last step, we use the optimal investment rule to compute the MRS of the capital decision maker as follows:

$$B_{ID(S)}(K, \Theta, e, e') = \beta \frac{u'\left[g_{ID(S)}^c(K', \Theta', e')\right]}{u'\left[g_{ID(S)}^c(K, \Theta, e)\right]}$$

where $K' = g_{ID(S)}^k(K, \Theta, e)$. Similarly, we can calculate the MRS for the other agent $i \neq ID(S)$ as follows:

$$B_i(K, \Theta, e, e') = \beta \frac{u'[g_i^c(K', \Theta', e')]}{u'[g_i^c(K, \Theta, e)]}$$

where the optimal capital is again determined by $K' = g_{ID(S)}^k(K, \Theta, e)$. The new MRS are used to update our initial guesses and to compute and update the price conjecture for every agent using (23), which has to satisfy the consistency condition for the capital decision maker:

$$P^C_{ID(S)}\left(K,\Theta,e,g^k_{ID(S)}\left(K,\Theta,e\right)\right)=P\left(K,\Theta,e\right) \tag{24}$$

Using the new price conjecture $P^C_{ID(S)}\left(K,\Theta,e,g^k_{ID(S)}\left(K,\Theta,e\right)\right)$ and the optimal capital $K'=g^k_{ID(S)}\left(K,\Theta,e\right)$, we can then update the continuation value in (22) using:

$$W_i\left(K,\Theta,e,g^k_{ID(S)}\left(K,\Theta,e\right)\right)$$

4.3 Numerical Results

In this section we present some numerical results from our two agent infinite horizon economy. To do this, we assume a Cobb Douglas production function $F\left(K,N\right)=K^{\alpha}N^{1-\alpha}$, with $\alpha=0.33$ and a CRRA utility function $u\left(c\right)=\frac{c^{1-\sigma}-1}{1-\sigma}$ with $\sigma=1$. The discount factor is set to $\beta=0.96$. As for the stochastic labor productivity shock e, we assume that it follows an AR(1) process:

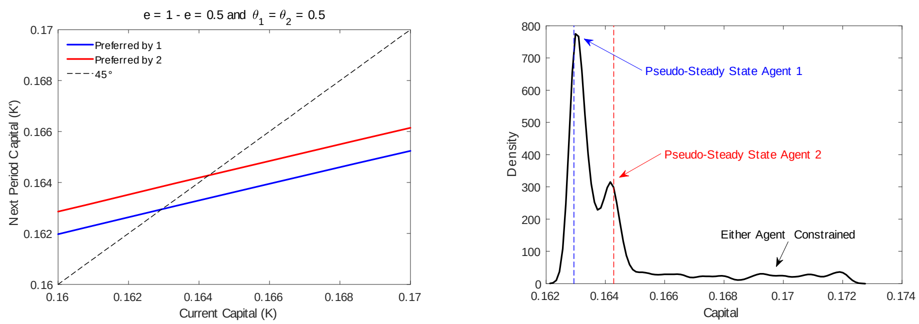
$$e'=c+\rho e+\varepsilon',\ \varepsilon'\sim N\left(0,\sigma^2_{\varepsilon}\right)$$

with $\rho=0.7$ and $\sigma_{\varepsilon}=0.05$ and $c=0.15$, which implies that the mean of the shock is 0.5. To compute our model, we discretize the process into a 9 state Markov chain. Finally, for the dividend taxes, we assume that they are equal to $\tau^1=0$ and $\tau^2=5\%$. Note that this is just an illustrative example but our aim is to highlight important features about how our model works that could be very important in a calibrated quantitative model with a reasonable wealth distribution.

Before we discuss the results from the simulations and policy functions, it is important to emphasize that, in our two agent model, the wealth-productivity distribution (Θ,e) does affect the law of motion of the aggregate capital stock K' due to the presence of binding borrowing constraints. In particular, in a higher joint dispersion of individual shareholdings and income shocks (low shares and low shocks or high shares and high shocks) will lead to a significantly higher capital accumulation compared to the case in which households have similar asset holdings and shocks. Intuitively, a more disperse wealth distribution means that low wealth agents are much closer to the no short selling constraint. In this case, market clearing requires a downwards adjustment of the return on capital (so that high wealth agents do not want to save as much), which can be achieved by increasing aggregate capital. These effects will be stronger when type 1 agents have low shares and income shocks or when they have high shares and income shocks (in which case the type 2 agents are close to the constraint). A different way of seeing this is the fact the borrowing constraints lead to precautionary savings motives that are higher the closer an agent is to the constraint, since the agent wants a buffer in case the constraint becomes binding. In our two agent model, this mechanism alone will lead to fluctuations in the aggregate capital even in the absence of aggregate uncertainty. In this paper, however, we provide an additional source of endogenous fluctuations in capital in the absence of aggregate uncertainty: the presence of shareholder disagreement. Since the two shareholders disagree on their preferred capital and the identity of the investment decision maker will change over time depending on who is the majority/median shareholder, capital will fluctuate when the identity of that shareholder even in the absence of the previous mechanism. We now illustrate this by looking at some of the resulting policy functions and simulations.

The left panel of Figure 1 below displays the policy functions for the preferred capital of the two agents as a function of the aggregate capital K for the average income shock and the same shareholdings. The right panel displays the density of capital over a long simulation.

Figure 1: Preferred Capital Policies and Optimal Capital Density

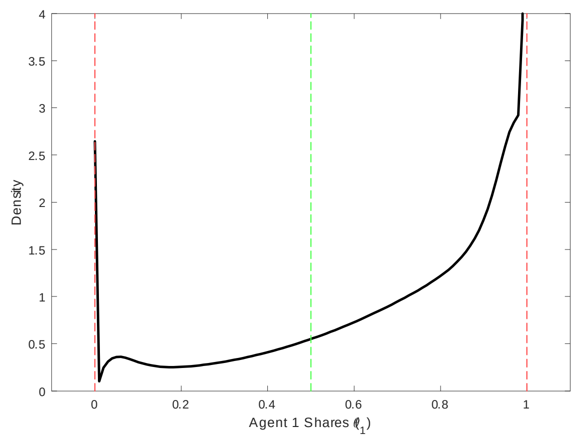


As in the three period model of the previous section, the left panel of Figure 1 reflects that the agent that is subject to the higher tax rate (agent 2) will want a higher aggregate capital for any level of initial capital K. Essentially, the intuition is the same as the one we provided in the three period model. Since the equity returns have to equalize across agents who hold a positive amount of shares, the agent with the higher tax rate will also have a higher MRS. Hence, the agent with a higher tax rate will want to transfer resources to next period, which he can do by investing more. Given this, the model has two "steady state" levels of capital, one for each agent type. As discussed earlier, this would not happen if taxes were the same across households. In that case, they would agree on the desired level of investment and capital would converge to a unique "steady state" with agreement.

The right panel of the figure displays the optimal capital density. As we see, capital is most of the time in the preferred "steady state" capital of agent 1, indicating that he is most of the time the investment decision maker. Capital is also often in the preferred "steady state" capital of agent 2, which is higher than that of agent 1. In addition, we see higher levels of capital much less often when one of the two agents' no short selling constraint is binding.

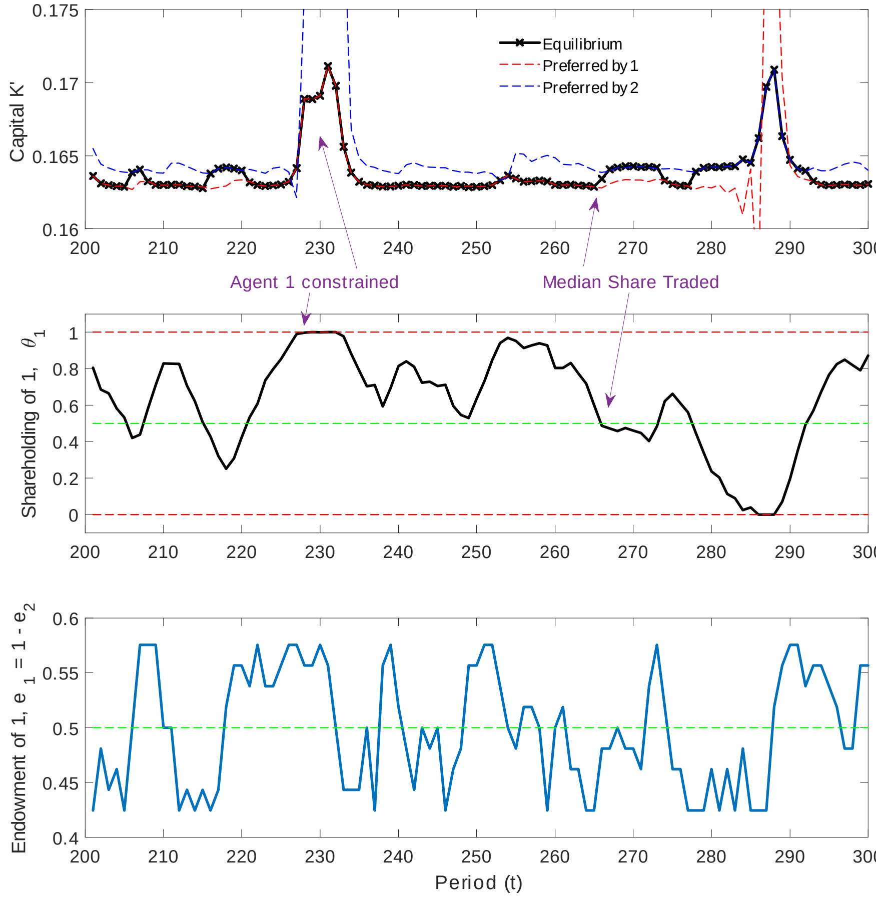
Figure 2 below displays the density for the shareholdings of agent 1 from a long simulation. As reflected by the figure, the density for the shares of agent 1 is more concentrated above one half, although there is also a non trivial mass below and specially at zero, which implies that the identity of the majority/median shareholder will switch and cause fluctuations in the optimal capital stock that are due to disagreement.

Figure 2: Density for Shareholdings of agent 1



The main findings just discussed by looking the the policy functions can also be observed in Figure 3 below, which displays simulated paths for the economy for 300 periods. The first panel of Figure 3 displays a simulation of the economy wide optimal level of capital, together with the preferred capital paths of agents 1 and 2. In addition, the second and third panels display the corresponding paths for the the shareholders and endowment shock of the first agent.

Figure 3: Simulated paths for Capital, Shareholdings and Endowment Shocks



As usual in these type of models, the endowment shocks and shareholdings are highly correlated, namely, the agent that gets the above average endowment shock is the one who will invest in shares. Thus, when an agent gets a sequence of above (below) average endowment shocks, his shares will keep increasing (decreasing) until the agent owns the whole firm (the agent hits the borrowing constraint). The simulation also confirms that the first agent is the one holding above average shares most of the time. This is reflected in the path for the optimal capital, which is most of the time the preferred capital of agent 1. We also see that capital stays very close to agent's 1 steady state level of capital except when his shareholdings approach one, implying that agent's 2 shares are close to the constraint. As discussed earlier, capital spikes up when an agent is close to the no short selling constraint to reduce the return and induce the other agent to save less. Finally, we also see an additional source of endogenous fluctuations in capital caused by the presence of shareholder disagreement when agents are far away from the borrowing constraints. In these periods, capital switches between the preferred steady state levels of the two agents every time the identity of the majority/median shareholder changes. Whereas the first source of fluctuations would not happen in a model with a continuum of agents and no aggregate risk in which shareholders are subject to the natural borrowing limit, in which case the wealth employment distribution would not affect the law of motion of aggregate capital, we would still have endogenous capital fluctuations caused by the presence of type specific dividend taxes leading to shareholder disagreement.

4.4 Extending the Algorithm to a Continuum of Agents

Ultimately, our goal is to compute the optimal capital in a model with many agents and progressive dividend taxation, since this will capture the wealth distribution and current tax system better than the two agent model with permanent tax differences in the previous section. While it is easy to incorporate progressive taxation into the previous model, it is more involved to extend our algorithm to be able to incorporate a large number of agents. In this case, two complications would arise. First, in a model with a large number of agents, the whole distribution becomes part of the aggregate state vector and we will have to approximate it with a finite number of moments for the model to be tractable computationally. Second, we need to extend the majority rule that we use with two agents to a setting with a large number of agents. In such a setting, each agent (type) can be characterized by their shareholdings and labor endowment at the start of the period and, given these variables, they decide on their shareholdings going forward. Each type will also have a preferred leval of capital and a measure in the population at the start of the period. In order to decide on the optimal economy wide capital stock, we can then order the agent types by their preferred level of capital. In equilibrium, we can then require that the level of capital chosen by the firm has the property that 50% of shares are held by types who want a weakly lower capital while the other 50% want a weakly higher capital. Note that this is the rule we have already used in the model with two agents, in which the median share (50% above vs below) will trivially be held by whichever agent happens to have over 50% of the shares.

5 Conclusion

TBA

6 References

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7 Appendix

7.1 Appendix A

In what follows, we provide details of the derivations of the investment optimality conditions in Section 3. To derive the optimality condition with respect to K_2 define the decision maker's value as $V_1^{ID_1*}(s_1) = u\left(c_1^{ID_1*}(s_1)\right) + \beta V_2^{ID_1*}(s_2)$, where $V_2^{ID_1*}(s_2) = u\left(c_2^{ID_1*}(s_2)\right)$ from the $t = 2$ solution. The optimality condition with respect to K_2 is $\frac{\partial V^*_{ID_1 1}(s_1)}{\partial K_2} = 0$, which can be writen as:

$$
\begin{aligned}
0 \quad = \quad & u^*_{c_1^{ID_1}}(s_1)\left[\left(1-\tau^{ID_1}\right)\frac{\partial d_1^*(s_1)}{\partial K_2^*(s_1)}\theta_1^{ID_1} + \frac{\partial p_1^{ID_1}(s_1)}{\partial K_2^*(s_1)}\left(\theta_{ID_1 1} - \theta_2^{ID_1*}(s_1)\right)\right] \\
& + \beta u^*_{c_{ID_1 2}}\left(1-\tau^{ID_1}\right)\frac{\partial d_2^*(s_2)}{\partial K_2^*(s_1)}\theta_2^{ID_1*}(s_1)
\end{aligned}
$$

$$
\begin{aligned}
\rightarrow \quad & 0 = u^*_{c_1^{ID_1}}(s_1)\left(\left(1-\tau^{ID_1}\right)\frac{\partial d_1^*(s_1)}{\partial K_2^*(s_1)} + \frac{\partial p_1^{ID_1}(s_1)}{\partial K_2^*(s_1)}\right)\theta_1^{ID_1} \\
& + \left[-u^*_{c_1^{ID_1}}(s_1)\frac{\partial p_1^{ID_1}(s_1)}{\partial K_2^*(s_1)} + \beta\left(1-\tau^{ID_1}\right)u^*_{c_2^{ID_1}}(s_2)\frac{\partial d_2^*(s_2)}{\partial K_2^*(s_1)}\right]\theta_2^{ID_1*}(s_1)
\end{aligned}
$$

and for $\frac{\partial p_1^{ID_1}(s_1)}{\partial K_2^*(s_1)}$ we use the conjecture $p_1^{ID_1}(s_1) = b_1 d_2(s_2) = b_1 A\left(K_2^*(s_1)\right)^{\alpha}$ so that

$$\frac{\partial p_1^{ID_1}(s_1)}{\partial K_2^*(s_1)} = b_1 \frac{\partial d_2^*(s_2)}{\partial K_2^*(s_1)} = b_1 \alpha A\left(K_2^*(s_1)\right)^{\alpha-1}$$

Consistency implies that $p_1^*(s_1) = p_1^{ID_1}(s_1)$ (i.e. the actual price equals the conjecture) can be used by replacing the actual price from the competitive equilibrium optimality condition and equating it to the conjecture to get:

$$b_1(s_1) = \beta\left(1-\tau^{ID_1}\right)\frac{u^*_{c_2^{ID_1}}(s_2)}{u^*_{c_1^{ID_1*}}(s_1)}$$

Note that b_1 is different depending on s_1.Replacing the derivative and equilibrium b_1 in the optimality condition for capital, we can see that the $\theta_2^{ID_1*}(s_1)$ term cancels and we are left with:

$$\begin{aligned} 0 &= u^*_{c_1^{ID_1*}}(s_1)\left(-\left(1-\tau^{ID_1}\right) + b_1 \alpha A\left(K_2^*(s_1)\right)^{\alpha-1}\right)\theta_1^{ID_1} \Rightarrow \\ 1 &= \beta \frac{u^*_{c_2^{ID_1}}(s_2)}{u^*_{c_1^{ID_1*}}(s_1)} \alpha A\left(K_2^*(s_1)\right)^{\alpha-1} \end{aligned}$$

Importantly, ID_1 depends on s_1 as described above ($ID_1 = 1$ if $\theta_1^1 \geq 0.5$ and $ID_1 = 2$ if $\theta_1^1 < 0.5$) and the marginal rate of substitution determining capital is different depending on who has the majority of the shares.

In what follows, we provide details of the derivation of the optimality condition with respect to K_1. Note that, from the $t=1$ solution $V_1^{i*}(s_1) = u\left(c_1^{i*}(s_1)\right) + \beta u\left(c_2^{i*}\left(K_2^*(s_1), \theta_2^{1*}(s_1)\right)\right)$ for $i = 1, 2$ (i.e. for both the investment decision maker and the other agent). So, for the decision maker at $t = 0$, we can write:

$$\begin{aligned} V_0^{ID_0*}(s_0) &= u\left(c_0^{ID_0*}(s_0)\right) + \beta V_1^{ID_0*}(s_1) = u\left(c_0^{ID_0*}(s_0)\right) + \beta V_1^{ID_0*}\left(K_1^*(s_0), \theta_1^{1*}(s_0)\right) \\ &= u\left(c_0^{ID_0*}(s_0)\right) + \beta\left[u\left(c_1^{ID_0*}(s_1)\right) + \beta u\left(c_2^{ID_0*}\left(K_2^*(s_1), \theta_2^{1*}(s_1)\right)\right)\right] \end{aligned}$$

where ID_0 is 1 if $\theta_0^1 \geq 0.5$ and 2 if $\theta_0^1 < 0.5$. The optimality condition implies:

$$\begin{aligned} 0 &= u_{c_0^{ID_0}}\left(\left(1-\tau^{ID_0}\right)\frac{\partial d_0^*(s_0)}{\partial K_1^*(s_0)} + \frac{\partial p_0^{ID_0}(s_0)}{\partial K_1^*(s_0)}\right)\theta_0^{ID_0} \\ &+ \left[-u_{c_0^{ID_0}}\frac{\partial p_0^{ID_0}(s_0)}{\partial K_1^*(s_0)} + \beta u_{c_1^{ID_0}}\left(\left(1-\tau^{ID_0}\right)\frac{\partial d_1^*(s_1)}{\partial K_1^*(s_0)} + \frac{\partial p_1^{ID_0}(s_1)}{\partial K_1^*(s_0)}\right)\right]\theta_1^{ID_0*}(s_0) \\ &+ \left[-\beta u_{c_1^{ID_0}}\frac{\partial p_1^{ID_0}(s_1)}{\partial K_1^*(s_0)} + \beta^2 u_{c_2^{ID_0}}\left(\left(1-\tau^{ID_0}\right)\frac{\partial d_2^*(s_2)}{\partial K_1^*(s_0)}\right)\right]\theta_2^{ID_0*}(s_1) \end{aligned}$$

We know that:

$$\begin{aligned}
\frac{\partial d_0^*(s_0)}{\partial K_1^*(s_0)} &= -1,\ \frac{\partial d_1^*(s_1)}{\partial K_1^*(s_0)} = \alpha A\left(K_1^*(s_0)\right)^{\alpha-1} - \frac{\partial K_2^*\left(K_1^*(s_0), \theta_1^{1*}(s_0)\right)}{\partial K_1^*(s_0)} \\
\frac{\partial d_2^*(s_2)}{\partial K_1^*(s_0)} &= \alpha A\left(K_2^*\left(K_1^*(s_0), \theta_1^{1*}(s_0)\right)\right)^{\alpha-1} \frac{\partial K_2^*\left(K_1^*(s_0), \theta_1^{1*}(s_0)\right)}{\partial K_1^*(s_0)}
\end{aligned}$$

Note that we still need $\frac{\partial p_1^{ID_0}(s_1)}{\partial K_1^*(s_0)}$, $\frac{\partial p_0^{ID_0}(s_0)}{\partial K_1^*(s_0)}$ and $\frac{\partial K_2^*\left(K_1^*(s_0), \theta_1^{i*}(s_0)\right)}{\partial K_1^*(s_0)}$. The first two derivatives will be based on consistent conjectures:

$$\begin{aligned}
p_1^{ID_0}(s_1) &= b_{12} d_2(s_2) = b_{12} A\left(K_2^*(s_1)\right)^{\alpha} \\
p_0^{ID_0}(s_0) &= b_{0,1} d_1(s_1) + b_{0,2} d_2(s_2) = b_{0,1}\left(A\left(K_1^*(s_0)\right)^{\alpha} - K_2^*(s_1)\right) + b_{0,2} A\left(K_2^*(s_1)\right)^{\alpha}
\end{aligned}$$

Using the previous conjectures, the derivatives are equal to:

$$\begin{aligned}
\frac{\partial p_1^{ID_0}(s_1)}{\partial K_1^*(s_0)} &= b_{12}\frac{\partial d_2^*(s_2)}{\partial K_1^*(s_0)} = b_{12}\alpha A\left(K_2^*\left(K_1^*(s_0), \theta_1^{i*}(s_0)\right)\right)^{\alpha-1} \frac{\partial K_2^*\left(K_1^*(s_0), \theta_1^{i*}(s_0)\right)}{\partial K_1^*(s_0)} \\
\frac{\partial p_0^{ID_0}(s_0)}{\partial K_1^*(s_0)} &= b_{0,1}\frac{\partial d_1(s_1)}{\partial K_1^*(s_0)} + b_{0,2}\frac{\partial d_2(s_2)}{\partial K_1^*(s_0)} \\
&= b_{01}\left(\alpha A\left(K_1^*(s_0)\right)^{\alpha-1} - \frac{\partial K_2^*(s_1)}{\partial K_1^*(s_0)}\right) + b_{0,2}\alpha A\left(K_2^*(s_1)\right)^{\alpha-1}\frac{\partial K_2^*(s_1)}{\partial K_1^*(s_0)}
\end{aligned}$$

Consistency implies that $p_1^*(s_1) = p_1^{ID_1}(s_1)$ and $p_0^*(s_0) = p_0^{ID_0}(s_0)$. At $t = 1$, the optimality condition for shares implies:

$$p_1^*(s_1) = \beta(1-\tau_i)\frac{u^*_{c_2^i}\left(K_2^*(s_1), \theta_2^{i*}(s_1)\right)}{u^*_{c_1^i}(s_1)} A\left(K_2^*(s_1)\right)^{\alpha} \text{ for } i = 1, 2$$

so that in principle we could find b_{12} using the after tax marginal rate of substitution of any of the two agents. If we use ID_1, then

$$b_{12}(s_1) = \beta\left(1-\tau^{ID_1}\right)\frac{u_{c_2^{ID_1}}(s_2)}{u_{c_1^{ID_1*}}(s_1)}$$

At $t = 0$, the optimality condition implies

$$p_0^*(s_0) = \beta\frac{u^*_{c_1^i}(s_1)}{u^*_{c_0^i}(s_0)}\left[(1-\tau_i) d_1^*(s_1) + p_1^*(s_1)\right] = \beta\frac{u^*_{c_1^i}(s_1)}{u^*_{c_0^i}}(1-\tau_i) d_1^*(s_1) + \beta\frac{u^*_{c_1^i}(s_1)}{u^*_{c_0^i}(s_0)} p_1^*(s_1)$$

Again, $p_1^*(s_1)$ could be expressed in terms of either agent's after tax marginal rate of substitution. If we use the after tax marginal rates of substitution of ID_0 and ID_1 respectively, we obtain:

$$p_0^*(s_0) = \beta\left(1-\tau^{ID_0}\right)\frac{u_{c_1^{ID_0}}(s_1)}{u_{c_0^{ID_0}}(s_0)} d_1^*(s_1) + \beta\frac{u_{c_1^{ID_0}}(s_1)}{u_{c_0^{ID_0}}(s_0)}\beta\left(1-\tau^{ID_1}\right)\frac{u^*_{c_2^{ID_1}}(s_2)}{u^*_{c_1^{ID_1*}}(s_1)} d_2^*(s_2)$$

which implies that:

$$b_{0,1}(s_0) = \beta\left(1-\tau^{ID_0}\right)\frac{u_{c_1^{ID_0}}(s_1)}{u_{c_0^{ID_0}}(s_0)},\ b_{0,2}(s_0) = \beta^2\left(1-\tau^{ID_1}\right)\frac{u_{c_1^{ID_0}}(s_1)}{u_{c_0^{ID_0}}(s_0)}\frac{u^*_{c_2^{ID_1}}(s_2)}{u^*_{c_1^{ID_1*}}(s_1)}$$

Instead, if we use the ID_0 agent's after tax marginal rate of substitution in the previous conditions, then

$$b_{0,2}(s_0) = \beta^2\left(1-\tau^{ID_0}\right)\frac{u_{c_2^{ID_0}}(s_2)}{u_{c_0^{ID_0}}(s_0)}$$

Any of these combinations should be equivalent. The crucial distinction will come when we use the investment optimality condition. Using the previous equations, the optimality condition can be expressed as follows. First, the term multiplying $\theta_2^{ID_0*}(s_1)$ becomes:

$$\begin{aligned}
& \left[-\beta u_{c_1^{ID_0}}\frac{\partial p_1^{ID_0}(s_1)}{\partial K_1^*(s_0)} + \beta^2 u_{c_2^{ID_0}}\left(\left(1-\tau^{ID_0}\right)\frac{\partial d_2^*(s_2)}{\partial K_1^*(s_0)}\right)\right] \\
= \quad & \left[-\beta u_{c_1^{ID_0}} b_{12} + \beta^2 u_{c_2^{ID_0}}\left(\left(1-\tau^{ID_0}\right)\right)\right]\frac{\partial d_2^*(s_2)}{\partial K_1^*(s_0)} = 0
\end{aligned}$$

and for b_{12} we can use either the ID_0 or the ID_1 after tax marginal rate of substitution since they are equalized. Either way, this term is zero. Second, for the term multiplying $\theta_1^{ID_0*}(s_0)$ we have:

$$\begin{aligned}
& \left[-u_{c_0^{ID_0}}\frac{\partial p_0^{ID_0}(s_0)}{\partial K_1^*(s_0)} + \beta u_{c_1^{ID_0}}\left(\left(1-\tau^{ID_0}\right)\frac{\partial d_1^*(s_1)}{\partial K_1^*(s_0)} + \frac{\partial p_1^{ID_0}(s_1)}{\partial K_1^*(s_0)}\right)\right] \\
= \quad & \left[-u_{c_0^{ID_0}}\left[b_{0,1}\frac{\partial d_1^*(s_1)}{\partial K_1^*(s_0)} + b_{0,2}\frac{\partial d_2^*(s_2)}{\partial K_1^*(s_0)}\right] + \beta u_{c_1^{ID_0}}\left(\left(1-\tau^{ID_0}\right)\frac{\partial d_1^*(s_1)}{\partial K_1^*(s_0)} + b_{12}\frac{\partial d_2^*(s_2)}{\partial K_1^*(s_0)}\right)\right] \\
= \quad & \frac{\partial d_1^*(s_1)}{\partial K_1^*(s_0)}\left[-u_{c_0^{ID_0}} b_{01} + \beta u_{c_1^{ID_0}}\left(1-\tau^{ID_0}\right)\right] + \frac{\partial d_2^*(s_2)}{\partial K_1^*(s_0)}\left[-u_{c_0^{ID_0}} b_{02} + \beta u_{c_1^{ID_0}} b_{12}\right] = 0
\end{aligned}$$

Both terms in the square brackets can be shown to be zero when the b's are replaced. So, we are only left with the term multiplying $\theta_0^{ID_0}$ term. This term can be rewriten as:

$$\begin{aligned}
& \left(\left(1-\tau^{ID_0}\right)\frac{\partial d_0^*(s_0)}{\partial K_1^*(s_0)} + \frac{\partial p_0^{ID_0}(s_0)}{\partial K_1^*(s_0)}\right) \\
= \quad & \left(\left(1-\tau^{ID_0}\right)\frac{\partial d_0^*(s_0)}{\partial K_1^*(s_0)} + b_{0,1}\frac{\partial d_1(s_1)}{\partial K_1^*(s_0)} + b_{0,2}\frac{\partial d_2(s_2)}{\partial K_1^*(s_0)}\right) \\
= \quad & -\left(1-\tau^{ID_0}\right) + b_{0,1}\left(\alpha A\left(K_1^*(s_0)\right)^{\alpha-1} - \frac{\partial K_2^*(s_1)}{\partial K_1^*(s_0)}\right) + b_{0,2}\alpha A\left(K_2^*(s_1)\right)^{\alpha-1}\frac{\partial K_2^*(s_1)}{\partial K_1^*(s_0)} \\
= \quad & -\left(1-\tau^{ID_0}\right) + b_{0,1}\alpha A\left(K_1^*(s_0)\right)^{\alpha-1} + \left(b_{0,2}\alpha A\left(K_2^*(s_1)\right)^{\alpha-1} - b_{0,1}\right)\frac{\partial K_2^*(s_1)}{\partial K_1^*(s_0)}
\end{aligned}$$

Note that the first term is the standard optimality condition for capital if the decision maker was always the agent that decides on capital at $t = 0$ ($ID_0 = ID_1$). The second term reflects the effect of a change in K_1 on p_0 through its effect on K_2 and it does not cancel out unless $ID_0 = ID_1$ because the K_2^* is chosen according to ID_1's marginal rate of substitution and not according to ID_0's. To see this note that:

$$\begin{aligned}
& b_{0,2}\alpha A\left(K_2^*\left(s_1\right)\right)^{\alpha-1} - b_{0,1} \\
= \quad & \beta^2\left(1-\tau^{ID_0}\right)\frac{u^*_{c_2^{ID_0}}\left(K_2^*\left(s_1\right),\theta_2^{i*}\left(s_1\right)\right)}{u^*_{c_0^{ID_0}}\left(s_0\right)}\alpha A\left(K_2^*\left(s_1\right)\right)^{\alpha-1} - \beta\left(1-\tau^{ID_0}\right)\frac{u^*_{c_1^{ID_0}}\left(K_1^*\left(s_0\right),\theta_1^{1*}\left(s_0\right)\right)}{u^*_{c_0^{ID_0}}\left(s_0\right)} \\
= \quad & \beta\left(1-\tau^{ID_0}\right)\frac{u^*_{c_1^{ID_0}}\left(K_1^*\left(s_0\right),\theta_1^{1*}\left(s_0\right)\right)}{u^*_{c_0^{ID_0}}\left(s_0\right)}\left[\beta\frac{MU^*_{ID_0 2}\left(K_2^*\left(s_1\right),\theta_2^{i*}\left(s_1\right)\right)}{u^*_{c_1^{ID_0}}\left(K_1^*\left(s_0\right),\theta_1^{1*}\left(s_0\right)\right)}\alpha A\left(K_2^*\left(s_1\right)\right)^{\alpha-1} - 1\right]
\end{aligned}$$

So the final condition is:

$$0 = \theta_0^{ID_0} u_{c_0^{ID_0}}\left[-\left(1-\tau^{ID_0}\right) + b_{0,1}\alpha A\left(K_1^*\left(s_0\right)\right)^{\alpha-1} + \left(b_{0,2}\alpha A\left(K_2^*\left(s_1\right)\right)^{\alpha-1} - b_{0,1}\right)\frac{\partial K_2^*\left(s_1\right)}{\partial K_1^*\left(s_0\right)}\right] \rightarrow$$